

Light is Heavy

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Abstract

Einstein's relativity theory appears to be very accurate, but at times equally puzzling. On the one hand, electromagnetic radiation must have zero rest mass in order to propagate at the speed of light, but on the other hand, since it definitely carries momentum and energy, it has non-zero inertial mass. Hence, by the principle of equivalence, it must have non-zero gravitational mass, and so, light must be heavy. In this paper, no new results will be derived, but a possibly surprising perspective on the above paradox is given.

Introduction

Einstein's general theory of relativity is based on two experimental facts. First, that the speed of light appears to be equal for all observers, independent of their velocity with respect to the light source, and second, the apparent equality of gravitational mass m_g and inertial mass m_i [1]. The latter is expressed by what is known as the principle of equivalence: "No experiment can distinguish the effects of a gravitational force from that of an inertial force in an accelerated frame". (Actually, only proportionality instead of equality between m_g and m_i is found, but the proportionality constant can be taken unity).

What are inertial mass and gravitational mass?

The inertial mass m_i is a measure of persistence to stay in the same state of motion, or like a resistance to acceleration, expressed by Newton's law $F = m_i a$. The inertial mass of an object can be determined, for example, by measuring the changes of velocities in a collision with another object of known mass. Gravitational mass m_g is a measure for attraction of and attraction by other masses M_g , it is like a "gravitational charge", the force being $F = GM_g m_g / r^2$. The gravitational mass of an object can be determined by putting it on a balance to compare it with a reference mass.

The paradox: light and heavy light

Electromagnetic radiation carries momentum $\vec{p} = \hbar\vec{k}$ and energy $E = \hbar\omega = pc$, and it can exert a (radiation) pressure on any object it falls upon. Hence, light has inertia that can be quantified by an inertial mass $m_i = \hbar k/c$. Now, the principle of equivalence tells us that light must also have a gravitational mass m_g , and consequently it must be attracted by heavy bodies. That this is the case is, of course, well known from the bending of starlight as observed during solar eclipse experiments as well as from the gravitational Doppler shift of light as seen in a vertical gamma-ray spectrometer employing the Mössbauer effect. If we would like to verify for electromagnetic radiation that indeed $m_g = m_i$, the question arises as how to determine the magnitude of its gravitational mass. For an arbitrary object, one would normally weigh the object, simply by putting it on a scale, at rest, thus measuring its rest mass m_0 . From this, it would appear that the rest mass and gravitational mass are the same thing. But how does one weigh light? It usually flies off with c , the speed of light! If, nevertheless, we would be able to accomplish this speedy task, we would find the mass of light to be zero. This can be seen as follows. Consider the Lorentz transformation of the inertial mass $m_i = \gamma m_0$ of an object with rest mass m_0 and velocity v (where $\gamma = 1/\sqrt{1 - v^2/c^2}$). In the limiting case of light-speed velocity, $v = c$, m_i becomes infinite unless $m_0 = 0$. Since light has finite inertia, the rest mass of light should be zero. This then seems to be in contradiction: on the one hand $0 \neq m_i = m_g$ and on the other $m_g = m_0 = 0$. The questions that emerge are: How to weigh anything properly? and: What is rest mass?

How to weigh a gas

How to properly weigh something as volatile as a gas? Simply put it in a box so that it doesn't fly away, and then put it on a symmetric balance. The reference mass should be put in a box of the same size to eliminate differences in up lift by the surrounding atmosphere. To make this plausible, consider, for example, a gas at temperature T and pressure P of N particles of mass $m = \gamma m_0$ inside a rectangular box of height h and volume $V = Ah$, with A the area of both its top and bottom. Due to gravitation, the pressure of the gas decays exponentially with height:

$$P(h) = P(h = 0)e^{-h\frac{mg}{kT}} \quad (1)$$

More gas particles collide faster with the bottom than with the top of the box, resulting in a pressure difference between the top and bottom ($h = 0$), given by $P(h) - P(h = 0) \approx -P(h = 0)hmg/kT$. Strictly speaking, this is only true if h is much smaller than the characteristic height $h_c = kT/mg$ which is, for example, 8.5 km for the atmosphere. For an ideal gas $P = kTN/V$ and the force is $F = PA$, hence the net force on the box is

$$F = -Nmg \quad (2)$$

Indeed, $M = Nm$ is what we would expect for both the inertial and the gravitational mass of the gas. Note that the mean velocity of the particles, and hence γ , increases with temperature. So, on weighing the box, we exactly do find the relativistic mass $M = Nm = N\gamma m_0$ of the gas, i.e. the rest mass of the particles plus the mass represented by their kinetic energy. The hotter the gas, the heavier the contents of the box. Fluctuations in pressure at time scales of the inverse collision frequency will be dampened/averaged out by the inertia of the balance such that a stable reading of the mass, the gravitational mass of the closed system, is obtained. Although nothing at all is at rest inside the box, the gravitational mass is equal to the rest mass of the box as a whole! The rest mass of the box is not equal to the sum of the rest masses of its contents. The reason that we find $M_g = M_0$ is that we have a closed system with the *centre of mass* of all the particles at rest.

Light on the balance

The same box, but now filled with light, and with the inner walls made perfectly reflecting, can be weighed too. Similar to the case where it was filled with gas particles, the light or, if you like it better, the photons, are gravitationally red or blue shifted at upward or downward propagation respectively. This again results in a net (radiation) pressure on the balance [2]. The shorter the wavelength, $\lambda = 2\pi\hbar c/E$, of the photons, the heavier the box. From the outside, it is *impossible* to judge whether the box is filled with a simple gas or with light. For a proof, compare the two drawings in Fig. 1, both showing light circulating with

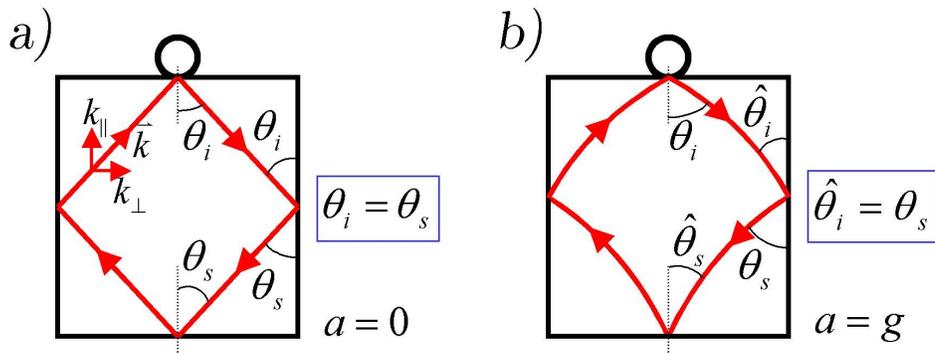


Figure 1: Light in a reflecting box, a) in free fall, b) in a strong gravitational field.

round trip time $\Delta t = t_{up} + t_{down} = 2t$ inside a reflecting box, where t is the time required to travel from top to bottom, or, because the speed of light is constant, vice versa. The first box is floating freely in space, the second is at rest on a heavy planet, or alternatively, is accelerated in deep space with $a = g$. In both cases, the observer and box are in the same frame. The wave vector \vec{k} and frequency ω of the light are related as follows: $k = \omega/c$,

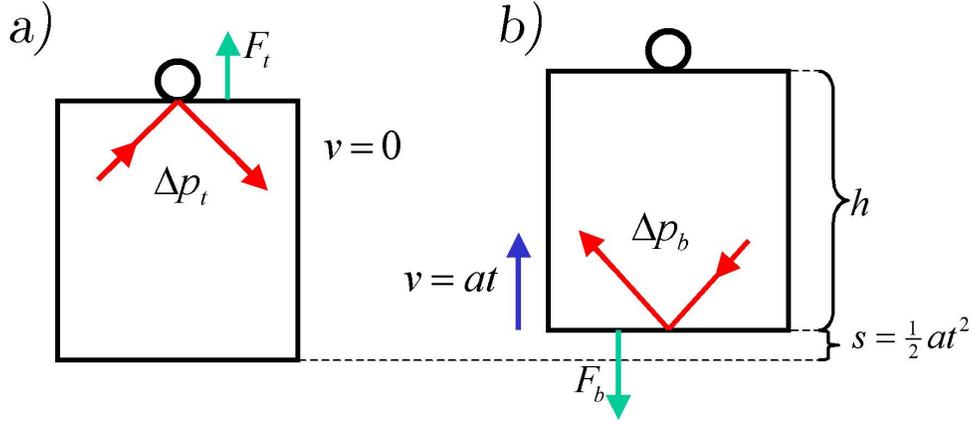


Figure 2: Snapshots of Fig. 1b, the momentum transfer at top and bottom, after application of the equivalence principle, both seen from the same frame.

$\cos \theta = k_{\parallel}/k$ and

$$k_{\parallel}^2 + k_{\perp}^2 - \frac{\omega^2}{c^2} = 0 \quad (3)$$

To calculate the net radiation force on the box, we will consider the reflection of a “photon” from top and bottom of the box separately, see snapshots of Fig. 1b in Fig. 2. The mass of the box is defined in the rest frame, $v = 0$, which we define with respect to be its point of suspension, the top, where we let the “photon” strike first. Now, we employ the equivalence principle on the bottom: we consider the gravitational force to come from an acceleration of the box upwards. The result is that the bottom seems to have velocity $v = at$ with respect to the top at the moment $t_{down} = t$ of impact of the “photon”. The Doppler shift of the light can be calculated using the following Lorentz transformations for k and ω in the rest frame to \hat{k} and $\hat{\omega}$ in a frame moving with velocity v :

$$\hat{k}_{\parallel} = \gamma \left(k_{\parallel} - \frac{v}{c^2} \omega \right) \quad (4)$$

$$\hat{k}_{\perp} = k_{\perp} \quad (5)$$

$$\hat{\omega} = \gamma \left(\omega - vk_{\parallel} \right) \quad (6)$$

From the above, it follows that we only need to consider those components of k which are (anti-)parallel to the acceleration. The total momentum transfer of light on top and bottom results in a net force:

$$\vec{F} = \vec{F}_b + \vec{F}_t = \frac{\Delta \vec{p}_b + \Delta \vec{p}_t}{\Delta t} \quad (7)$$

The momentum transfer during one roundtrip is calculated in the momentary frame of the box (for downward flight $k_{\parallel} \rightarrow -k_{\parallel}$):

$$\Delta p_t = 2\hbar k_{\parallel} \quad (8)$$

$$\Delta p_b = \frac{2}{\gamma} \hat{p}_b = \frac{2}{\gamma} \hbar \hat{k}_{\parallel} = 2\hbar \left(-k_{\parallel} - \frac{v}{c^2} \omega \right) \quad (9)$$

Substituting those in Eq. (7), and using that $\Delta t = 2t$, $v = at$, $E = \hbar\omega$ and $a = g$ we find that

$$F = -\frac{\hbar\omega}{c^2} \frac{2v}{\Delta t} = -\frac{E}{c^2} a = -m_g g \quad (10)$$

which concludes the proof that the gravitational mass of light is $m_g = E/c^2$. See also Ref. [2].

Guess who?

A combination of both the gas and light examples presented above is offered by the dramatic event of electron-positron pair annihilation. In the simplest case, just two photons are produced. Matter is fully transformed to radiation, but the mass stays. Put on a balance in a box, it is impossible to know whether or not the pair has decayed. This example shows that the equation $E = mc^2$ expresses the equivalence of mass and energy and *not* the generation of energy as a reaction product from mass. The confusion that sometimes arises can often be traced back to the mix-up between the words “mass” and “matter”. Matter can be transformed into radiation. Matter is taking the role of energy container, radiation is some sort of released, “free” energy, that must fly through space.

Discussion

In the case of light, the rest mass is zero, but the gravitational mass equals the inertial mass, which is identical to the relativistic mass. The “photon” can only be weighed if it is contained in one way or another, so that its centre of mass is fixed (on average). In case we weigh any material object, heat, rotational, vibrational and kinetic energy, the sort of energy naturally contained in matter, put their weight to the scale. It shows that the term “rest mass” really only means that the centre of mass of the object is at rest in the frame of the observer. We can think of material objects as being built out of some smaller constituents, glued together by some binding force. We go from houses to bricks, from bricks to molecules, from molecules to atoms, from atoms to nucleons and electrons, and from there to quarks and still electrons (we could have started from cosmic super clusters). From this list it should dawn on us that, every time we think, at first glance, that we are dealing with a rigid chunk of matter (planet, brick, atom), it appears to carry a lot of dynamics at various length scales and energies. The smaller the length scales, the stronger the forces involved and the higher the (binding) energies, and hence the corresponding masses, relative to the rest masses of the constituents. We could wonder whether this finds its climax at a point where an elementary material particle is built of constituents that have zero rest mass, with only kinetic and potential energy to make up for its mass. That this should be the case for the electron, but at the same time seems quite impossible [3], is well known [4]. What is intriguing is that matter’s most basic building blocks, the elementary particles, all

have non-zero spin, intrinsic angular momentum, which seems to imply that they all must have some sort of intrinsic dynamics. Hypothetical structures which do not have internal dynamics, such as point particles and hard spheres, do not exist. So what is matter really made of then? In the Dirac theory, the electron is like electromagnetic energy quivering at light speed, just like a photon in a box [5]. If really so, matter is light.

Conclusions

- Rest mass never applies to a system at complete rest, because such systems do not exist; there will *always* be internal dynamics.
- Rest mass applies to the centre of mass of a closed system
- The gravitational mass is equivalent to the total energy of an object or system.
- The mass of a closed system is always conserved. This is just the energy conservation law rephrased. Mass and energy are equivalent.

One could say: “Matter is just “canned” energy, a box with internal dynamics, and radiation is “free” energy.” If the photon would be put to rest, its gravitational mass would equal its rest mass, and hence vanish. The intriguing question is, what would happen if we could stop the electron from spinning?

References

- [1] Bertram Schwarzschild, “Gravitational Self-Energy and the Equivalence Principle”, *Phys. Today* **Nov.**, 19 (1999).
- [2] R.P. Feynman, R.B. Leighton, M. Sands, “Curved Space”, *The Feynman Lectures on Physics* (commemorative issue), Vol. **II**, Chap. 42, (Addison-Wesley, Reading, 1989); *ibid.* **Vol. I**, Chap. 7, 15, 16.
- [3] R.P. Feynman, R.B. Leighton, M. Sands, “Electromagnetic Mass”, *The Feynman Lectures on Physics*, Vol. **II**, Chap. 28, (Addison-Wesley, Reading, 1989).
- [4] Frank Wilczek, “Mass without Mass I: Most of Matter”, *Phys. Today* **Nov.**, 11 (1999); “Mass without Mass II: The Medium is the Mass-age”, *Phys. Today* **Jan.**, 13 (2000).
- [5] J.G. Williamson and M.B. van der Mark, “Is the electron a photon with toroidal topology?”, *Ann. Fondation L. de Broglie* **22**, 133 (1997); M.B. van der Mark and J.G. Williamson, “Charge as a pure field configuration”, draft.