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Quantum particle, light clock or heavy beat box?

Martin B van der Mark

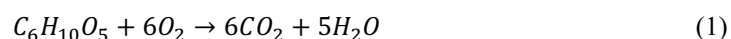
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Abstract. A standing wave of light inside a cavity, while observed from a moving frame, is seen to have a spatial beat of precisely the same nature as that of the de Broglie wave of a quantum particle. The structural forces that hold together the cavity against the internal radiation pressure are compared to the so-called Poincaré stresses inside an elementary particle, as are required for the particle's stability. This seems to work best if the cavity is non-rigid and hence is itself oscillating, coupled to the radiation inside.

1. Introduction

What is a plank? It is a rectangular object made of some stuff called wood and it has a porous and fibrous structure, a composite of fibers embedded in a matrix of lignin. The fibres are made of cellulose, a polysaccharide with the formula $(C_6H_{10}O_5)_n$ that consists of a linear chain of rings of D-glucose. We can find out a lot about the wood's structure by looking at it under an (electron) microscope and we can find out about its composition by chemical analysis, for example by burning it in a controlled way and looking at the amount of products; water and carbon dioxide in this case:



We may look even deeper and realize that each of the atoms in the wood consist of a nucleus of protons and neutrons with an electron cloud surrounding it. Then, looking even deeper, we find that the electrons are spinning and waving, and that they are made of...? We do not know what it is! Maybe we need to "burn" some of them to find out. Already, nature presents us with an interesting clue:



The two photons that are produced in electron-positron annihilation are suggestive, but certainly not conclusive about the stuff that may constitute both the electron and positron [1,2]. Nonetheless it appears to be very instructive to look at a photonic structure that may, if only in some respects, serve as a particle model [2]. Even before we may touch upon the structure of particles, the deeper nature of space, time and the vacuum forms a further complication in our understanding of what is going on around us.

In this paper a very simple oscillating system will be studied, a so-called light clock, when it is observed from a boosted frame. It is shown how the beat of the internal wave of a light clock and the de Broglie wavelength of a quantum particle may have the same origin. If, in some way, elementary particles would be confined photons, as in the light clock, this implies what should be the role of the



photon and the nature of the cavity. Consequently, some light is shed on the possible nature of elementary matter, its internal binding forces (the so-called Poincaré stresses [3]) and the vacuum.

2. Clockwork and relativity

A clock in relative motion to its observer will be studied. Basically, a clock is a rather simple device that may consist of only the following basic elements:

- an oscillator
- an energy source (just to keep it going)
- a counter (just to memorize the time (number of oscillations) that passed)

The essence of the clock is only in its oscillator, not a *time* but a *frequency* generator, which itself consists of two elements:

- a resonator
- something vibrating, subject to the resonator

For example, an oscillator may consist of a suspended weight with a restoring force acting on it, such as a pendulum in the gravitational field or a balance with spring. Also it may be of electro-mechanical origin employing a piezo-electric crystal. Yet other types of clocks exists, such as the hourglass, which work according to quite different principles.

2.1. The light clock

In this paper we will focus on what is in some respects the simplest clock imaginable: the light clock. As is shown in figure 1, the light clock consists of two mirrors with light reflected in between and it has the following properties:

- A single pulse of light travels up and down the resonator, with period: $P = 2L/c$
- If the resonator period is shorter than the duration of the pulse, a standing wave is formed with $L = n\lambda/2$, because the wavelength must fit to the resonator cavity

For our aim we do not need any hands to memorize and read the time that passed.

Apparently, from the two properties above, it is found that there is one resonance condition for space, connecting the cavity size L , to the wavelength λ and another resonance condition for time, connecting the cavity size L to the cavity period P as well. This no rocket science, but its subtlety may still show some surprises later on.

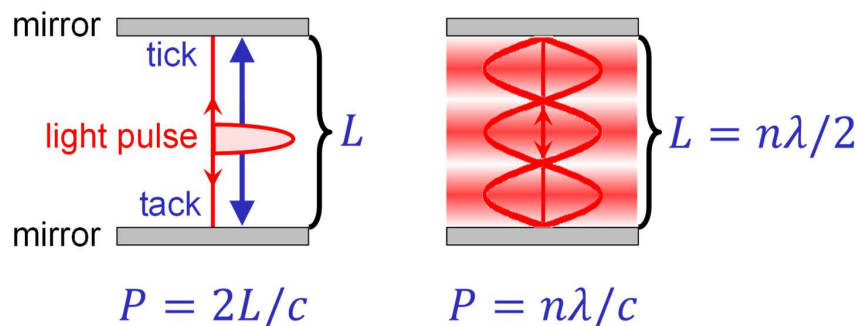


Figure 1. The light clock consists of two mirrors (the resonator) and a pulse of light traveling in between (**left**). A standing wave is formed if the light wave between the mirrors is continuous (**right**).

2.2. The light clock in motion

We want to look what happens when we put the clock in motion. Better, to prevent any suggestion of accelerations that may disturb the clocks integrity, the observer is boosted in the opposite direction, see figure 2. Note that the following standard definitions will be used:

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c} \tag{3}$$

On the one hand, time dilatation is seen when the clock is observed from a fast moving frame. The clock is seen to be slowing down. The path over which the light travels is seen longer because it zig-zags while the speed of light remains the same constant c , hence the resonator (clock) period increases.

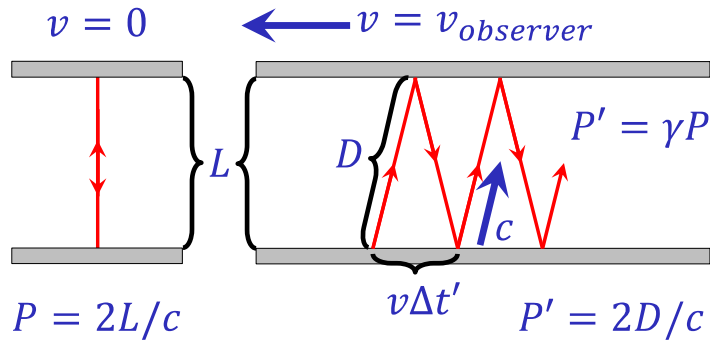


Figure 2. The clock seen from the same frame (stationary) (left). The clock as seen from a fast moving frame. The light path of the pulse is seen longer because it zig-zags while the speed of light remains constant, hence the resonator (clock) period increases (right).

On the other hand, there is a Doppler effect due to the Lorentz transformations:

$$\vec{k}' = \vec{k} + (\gamma - 1)(\vec{k} \cdot \vec{v})\vec{v}/v^2 + \gamma\omega\vec{v}/c^2 \tag{4a}$$

$$\omega' = \gamma(\omega + \vec{k} \cdot \vec{v}) \tag{4b}$$

In the present example it was deliberately chosen that the boost is at right angles with the direction of motion of the light in the cavity, just to make things simple. Later we will show that the general case has no further surprises. The result is that there is a blue shift of the light only due to the so-called transverse Doppler effect, see figure 3.

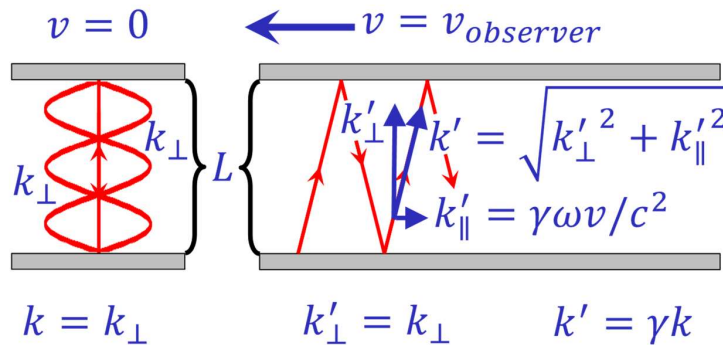


Figure 3. In the stationary frame the cavity has a standing wave with $k = k_{\perp}$ (left). From the boosted frame, a blue shift of the light is observed due to the transverse Doppler effect, $k' = \gamma k$ and $\omega' = \gamma\omega$ (right).

The transverse Lorentz transformations, for which $k_{\parallel} = 0$, $k = k_{\perp}$, can be obtained from the general transformations:

$$k'_{\perp} = k_{\perp}, \quad k'_{\parallel} = \gamma\omega v/c^2, \quad (5a)$$

$$\omega' = \gamma\omega \quad (5b)$$

We further recall that $\omega = kc$ and $\omega' = k'c$. In the boosted frame, a component of the wave vector appears that is parallel to the direction of motion. The orthogonal component of the wave vector and hence the resonance of the cavity are unaffected. Hence, the total wave vector is blue shifted: $k' = \gamma k$.

From figures 2 and 3, we have two effects:

- The clock frequency $f = 1/P$ decreases: $P' = \gamma P$, hence $f' = f/\gamma < f$
- The light frequency $\omega = kc$ increases: $k' = \gamma k$, hence $\omega' = \gamma\omega > \omega$

Where one frequency is related to the cavity, the resonator, and the other frequency is related to the internal wave (the vibrating thing). One frequency, ω , goes up while the other, f , goes down. How then do they keep in synchrony? Well, according to moving observers they don't, but despite this, consistency is not affected, as is shown below.

2.3. On the beat of the light clock

Whenever we add (as in linear superposition of waves) two notes of slightly different frequencies $\omega - \delta$ and $\omega + \delta$, a beat will appear between them:

$$\cos(\omega - \delta)t + \cos(\omega + 2\delta)t = 2 \cos\left(\frac{\omega - \delta - \omega + \delta}{2}t\right) \cos\left(\frac{\omega - \delta + \omega + \delta}{2}t\right) = 2 \cos \delta t \cos \omega t$$

What we have is the product of the average frequency ω and half the difference frequency δ , the latter in effect slowly modulating, by $\cos \delta t$, the note's amplitude, which we call the "beat".

Similarly, inside the light clock there is a superposition of two counter-propagating waves, together forming a standing wave:

$$\cos(\omega t - kx) + \cos(\omega t + kx) = 2 \cos(\omega t) \cos(kx) \quad (6)$$

With a boost in the z -direction and $k = k_{\perp}$ this yields:

$$\cos(\omega't - k'_{\perp}x - k'_{\parallel}z) + \cos(\omega't + k'_{\perp}x - k'_{\parallel}z) = 2 \cos(\omega't - k'_{\parallel}z) \cos(k'_{\perp}x) \quad (7)$$

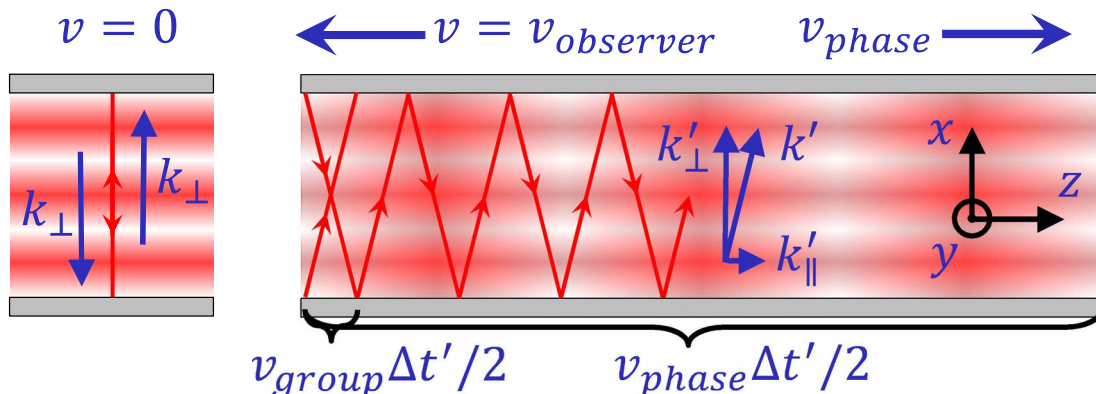


Figure 4. In the stationary frame, the cavity only shows a standing wave. In the boosted frame this standing wave remains but on top of that a spatial beat appears, like that of a Moiré pattern, which is propagating in the z -direction at the phase velocity v_{phase} .

In the left panel of figure 4, the cavity with its standing wave is shown in the stationary frame. The right panel shows the situation as seen from the boosted frame, where the vertical standing wave remains, $k'_\perp = k_\perp$ but now with a horizontal spatial beat appearing on top of that: $k'_\parallel = \gamma\omega v/c^2 = \gamma v k_\perp/c$. This spatial beat is propagating in the z-direction at phase velocity:

$$v_{phase} \equiv \frac{\omega'}{k'_\parallel} = \frac{c^2}{v} \quad (8)$$

Note that $\vec{v}_{observer} = -\vec{v}_{group}$ and $v = v_z = |v_{observer}| = |v_{group}|$. The wavelength of the beat is

$$\lambda_{beat} = \frac{2\pi}{k'_\parallel} = \frac{2\pi c^2}{\gamma v \omega} \quad (9)$$

For ease of interpretation equation 7 may be rewritten as follows:

$$\cos(\omega't - k'_\parallel z) \cos(k'_\perp x) = \cos(\omega_{phase}t - 2\pi z/\lambda_{beat}) \cos(kx) \quad (10)$$

where $\omega_{phase} \equiv \omega' = \gamma\omega$.

Hence it is found that a boosted light clock beats in *space, not in time*. If the cavity is very long in the z-direction, and the observer would be moving inside, he would experience an undulation in space, propagating towards him at the phase velocity.

2.4. Some remarks on mass and energy

In this paper, so far, it has been taken that the mirrors of the clock cavity always stay at the same distance, in other words, the cavity has been assumed rigid. Further, the clock cavity is taken to be massless.

If the light in the cavity consists of j photons, their momentum is $\vec{p} = j\hbar\vec{k}$ and they will exert a radiation pressure on the cavity mirrors. Localized inside the clock, each photon reflects its momentum on the mirrors, and in this way each photon expresses its inertial mass. Consistent with its momentum, each photon has energy $E = \hbar\omega = \hbar kc$ and with it comes a mass according to $E = mc^2$, hence

$$m_{photon} = \hbar\omega/c^2 \quad (11)$$

consistent with the Compton effect.

Sometimes the above seems to puzzle even the most knowledgeable of scientists. This is because we have always been told that the rest mass m_0 of a photon is zero: $m_0 = 0$. This is not wrong. The problem of course is that light, and the photon, are never at rest but instead are always moving at light speed. The statement that "the rest mass of light is zero" has no direct physical meaning! The rest mass of light does have a "limited" meaning, however, in the sense that:

$$\lim_{v \rightarrow c} \gamma m_0 = m_{photon} \quad (12)$$

with $m_{photon} = \infty$ unless $m_0 = 0$, in which case m_{photon} is finite. For the record, light does not only have inertial mass, it is bend in gravitational fields and hence must have gravitational mass too [4]: "Since energy E has gravitational mass E/c^2 the photon has a mass (not a rest mass) $\hbar\omega/c^2$ and is attracted by the earth." [5]. Indeed, the mass m_{photon} is exactly what we weigh for photons in a box [5,6].

2.5. The (single) photon clock

If the light wave in the clock consists of a number of j photons of wavelength λ , the total energy in the cavity is $E = j\hbar\omega = jhc/\lambda$. This energy corresponds to a mass:

$$M_{light} = jm_{photon} = \frac{E}{c^2} = \frac{j\hbar}{\lambda c} \quad (13)$$

The total rest mass of the light clock is

$$m_0 = m_{cavity} + M_{light} \quad (14)$$

For a single photon inside the clock, $j = 1$, and $m_{cavity} = 0$ it is found that the wavelength of the light in the cavity is equal to, and can be interpreted as, the Compton wavelength λ_C of a particle of mass m_0 :

$$\lambda = \frac{j\hbar}{M_{light}c} = \frac{\hbar}{m_0c} \equiv \lambda_C = \frac{2\pi c}{\omega_C} \quad (15)$$

Again, the essential thing that has been put in is that the mass m_0 corresponds to the energy of a single photon, $m_0c^2 = \hbar\omega_C$ with $\omega = \omega_C$ the Compton frequency. Note that this not an extra assumption, it is how the quantum \hbar always enters quantum mechanics.

It is now possible to interpret further the beat of the light clock in terms of the photonic quantization of light energy. In equation 9 we found the wavelength of the clock beat

$$\lambda_{beat} = \frac{2\pi}{k'_\parallel} = \frac{2\pi c^2}{\gamma v \omega} = \frac{\hbar}{\gamma m_0 v} \equiv \lambda_B = \frac{2\pi}{k_B} \quad (16)$$

With photon quantization taken into account we see that the wavelength of the internal beat of the moving clock is *identical* to the de Broglie wavelength λ_B for a particle of mass m_0 . The de Broglie wave vector $\vec{k}_B = \vec{k}'_\parallel$ is pointing in the direction of the boost. The beat length is independent of the number of photons in the cavity which implies that each of them forms an independent clock within the same rigid cavity.

2.6. The boosted light clock at arbitrary orientation

So far, a special orientation of the light clock was chosen. The question is, does the previous result depend critically on the orientation of the clock with respect to the direction of the boost? Figure 5 shows the two counter-propagating waves forming a standing wave in a general direction:

$$\cos(\omega t - \vec{k} \cdot \vec{r}) + \cos(\omega t + \vec{k} \cdot \vec{r}) = 2\cos(\omega t) \cos(\vec{k} \cdot \vec{r}) \quad (17)$$

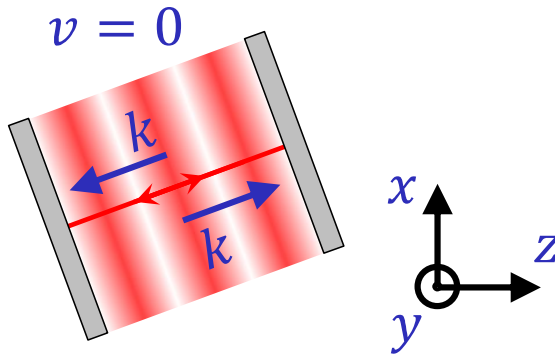


Figure 5. The light clock at arbitrary angle to the z-direction

The Lorentz transformations, with $k = k_C$ are:

$$\vec{k}^\pm = \vec{k} + (\gamma - 1)(\vec{k} \cdot \vec{v})\vec{v}/v^2 \pm \gamma\omega\vec{v}/c^2 \equiv \Gamma\vec{k} \pm k_B\vec{v}/v \quad (18a)$$

$$\omega^\pm = \gamma(\omega \pm \vec{k} \cdot \vec{v}) = k_B v_{phase} \pm \gamma\vec{k} \cdot \vec{v} \quad (18b)$$

where $v = v_z$ and $v_{phase} = c^2/v$ and $\vec{k}_B = k_B\vec{v}/v = \gamma\omega\vec{v}/c^2$

The general standing wave with boost $v = v_z$ in the z-direction:

$$\cos(\omega^+ t - \vec{k}^+ \cdot \vec{r}) + \cos(\omega^- t + \vec{k}^- \cdot \vec{r}) = 2 \cos(k_B z - \omega_{phase} t) \cos(\Gamma \vec{k} \cdot \vec{r} - \gamma \vec{k} \cdot \vec{v} t) \quad (19)$$

This shows that the de Broglie beat: $\cos(k_B(z - v_{phase} t)) = \cos(k_B z - \omega_{phase} t)$ is independent of the direction of motion with respect to the standing (plane) wave inside the Fabry-Pérot cavity. A sum of plane waves of all directions and wavenumbers can make a wave of any shape. This implies that the de Broglie beat is a property of *any* mode inside any cavity of *arbitrary* shape, and one of the more important messages of this paper. The beat propagates at phase velocity $v_{phase} = c^2/v$ and is internal to the clock cavity. Consequently, if the cavity is closed and rigid, the beat will be unobservable external to the cavity.

3. Relativistic matter and waves

According to Einstein, in special relativity, mass, energy and momentum are related as follows:

$$E = mc^2 = \gamma m_0 c^2 = \sqrt{m_0^2 c^4 + p^2 c^2} \quad (20)$$

According to Planck the energy and momentum of the photon are $E_0 = \hbar\omega$ and $\vec{p} = \hbar\vec{k}$, and Louis de Broglie imagined [7] that, “by cause of a meta law of nature, to each portion of energy with proper mass m_0 , one may associate a periodic phenomenon of frequency ω_C ”, such that if all energy quanta have the same dependence on frequency as the photon, the following must hold:

$$\hbar\omega_C = m_0 c^2 \quad (21)$$

And this is how quantum mechanics was born from special relativity and black-body radiation. With $\omega = \gamma\omega_C$ and $|\vec{k}| = k = k_B$ we can write:

$$E = \gamma\hbar\omega_C = \sqrt{\hbar^2\omega_C^2 + \hbar^2 k_B^2 c^2} = \hbar\sqrt{\omega_C^2 + \omega_B^2} \quad (22)$$

where $k_B c = \omega_B$ and $\omega_B = \gamma v \omega_C / c$, consistent with equations 15 and 16. The group- and phase velocities are:

$$v_{phase} \equiv \frac{\omega}{k} = \frac{\omega_{phase}}{k_B} \quad (23)$$

$$v_{group} \equiv \frac{\partial\omega}{\partial k} = \frac{\partial\sqrt{\omega_C^2 + k_B^2 c^2}}{\partial k} = \frac{2k_B c^2}{2\omega} = \frac{c^2}{v_{phase}} \quad (24)$$

It appears that in quantum mechanics $v = v_{group} = v_{particle}$, consistent with equation 8, and that the frequency belonging to the wave vector $|\vec{k}| = k_B$ is $\omega = \omega_{phase} = \gamma\omega_C$

3.1. The harmony of phases

Louis de Broglie realized that a particle with a quantum of energy, associated with a periodic phenomenon, as he postulated, must show the same paradox as described for our light clock described in chapter 2. When boosted, the periodic phenomenon must slow down while the mass of the particle, hence the frequency, should go up! He solved this by postulating an additional wave that is associated with each particle, the wave being in phase with the periodic phenomenon at all times. He denoted this as “the theorem of phase harmony”, and realized that the wave represented a spatial distribution of phase, propagating at velocity $v_{phase} = c^2/v$. In his thesis [7], equation (1.2.1) represents the same kind of beat that we have in equation 7, but without a physical representation of the light-speed standing wave (the photon, representing the quantized periodic phenomenon) inside a cavity (to trap the photon

and express its mass). In our interpretation, photon and cavity together make the light clock (representing the particle).

Although de Broglie fixed the paradox brilliantly and therewith spawned modern quantum mechanics, he left us with an unexplained *reason* for the wave-particle duality. He repaired this, but only in the space surrounding a point particle, with his double solution theory [8] and the pilot wave picture, with David Bohm. Would he have used a light clock analog for his solution to pinpoint the underlying “meta law of nature” of the “periodic phenomenon” as mentioned earlier, the wave-particle duality and the true nature of elementary quantum particles would by now have understood differently, perhaps. Before discussing this point, it is important to look a bit further at the properties of the light clock and the de Broglie wave.

The quantum particle, according to de Broglie, has a basic wave number $k_C = \omega_C/c$ and a single *external* wave vector \vec{k}_B , the de Broglie wave vector accompanying the particle, which is related to two frequencies and waves propagating at different velocities v, v_{phase} and c :

$$k_B v = \omega_{group} \quad \varphi_v = \omega_{group} t_v - k_B z \quad z = v t_v \quad (25a)$$

$$k_B v_{phase} = \omega_{phase} \quad \varphi_\varphi = \omega_{phase} t_\varphi - k_B z \quad z = v_{phase} t_\varphi \quad (25b)$$

That both waves have the same wave vector, ensures the harmony of phases because the wave velocity and frequency have the same ratio for both waves.

The massless, rigid light clock also has a basic wave number $k_C = \omega_C/c$ but now with a single *internal* wave vector \vec{k}_B , which is related to the same two frequencies and waves here above, plus a third, light-speed wave that is also in harmony:

$$k_B c = \omega_B \quad \varphi_c = \omega_B t_c - k_B z \quad z = c t_c \quad (25c)$$

where $\omega_B^2 = \omega_{group} \omega_{phase}$, in other words the light clock shows what we will call a *triple harmony*. For the light clock, the following frequencies are found:

- The frequency of the quantum mechanical wave:
$$\omega = \gamma \omega_C = \omega_{phase} = k_B v_{phase}$$
- The beat frequency between the quantum phase wave and the periodic phenomenon:
$$\omega_{beat} = \omega_{wave} - \omega_{clock} = \gamma \omega_C - \omega_C / \gamma = \beta^2 \gamma \omega_C = \omega_{group} = k_B v$$
- An intermediate frequency that we may call the de Broglie frequency:

$$\omega_B = \gamma \beta \omega_C = k_B c$$

Given the presence of the difference frequency $\omega_{group} = \gamma \omega_C - \omega_C / \gamma$, there may be a sum frequency

$$\omega_Z = \gamma \omega_C + \omega_C / \gamma$$

hidden somewhere as well, which would imply some kind of a nonlinearity in the system. The difference frequency is only there as a result of the nonlinearity of the Lorentz transformation. At zero particle velocity ($v = 0$) it naturally vanishes, $\omega_{group} = 0$, but the sum frequency $\omega_Z = 2\omega_C$ would still be there, at least for now, without any obvious nonlinearity to support it. The reason why to speculate on its possible existence is that $\omega_Z = 2\omega_C$ reminds us of the so-called Zitterbewegung frequency of a spin- $\frac{1}{2}$ particle [2,9].

With the results obtained in this paper so far, it is now time to compare and discuss a few properties of the light clock and a quantum particle according to de Broglie.

4. Discussion

That a light clock may be a suitable guiding model for the both the external and internal structure of an elementary quantum particle may seem ridiculous. The main result so far is that a light clock provides an internal structure that naturally leads to a de Broglie wave. The light clock model makes explicit, on the one hand, the wave-particle behavior, and on the other hand what an elementary quantum system is suggested to be made of, namely confined electromagnetic waves. However, if the model of a light clock

is going to be any good for describing the basic nature of an elementary quantum particle, a few problems need to be solved:

- What is the nature of the clock cavity, what is its mass, and how does it relate to the binding forces (known as the Poincaré stresses) inside elementary particles?
- How can the internal standing wave of the light clock express itself to the outside?

Then there is also a question from the point of view of the quantum particle:

- What is the Zitterbewegung, and how may this be embedded in a light clock?

The assumed model of the light clock has been that its cavity is rigid and massless, with a standing wave electromagnetic wave inside. A rigid cavity presumably has sturdy struts that hold the mirrors together. Binding forces are required to make the struts rigid, even before any external force acts on them. These binding forces represent energy, and energy is equivalent to mass. This then contradicts the assumption that the cavity is massless and that the total mass of the light clock is only determined by the energy of the photon. A next step in our analysis should be to include some more realistic properties for the clock cavity.

4.1. The light clock with a more physical cavity

What if the cavity of the light clock is made elastic, with the mirrors held together by springs? In figure 6, from top to bottom, four moments in time during one oscillation period P are shown. A light pulse is propagating up and down the cavity and its radiation pressure $\Delta p = 2\hbar k = 2\hbar\omega/c$ will make the distance between the mirrors oscillate as if it were breathing like a beat box.

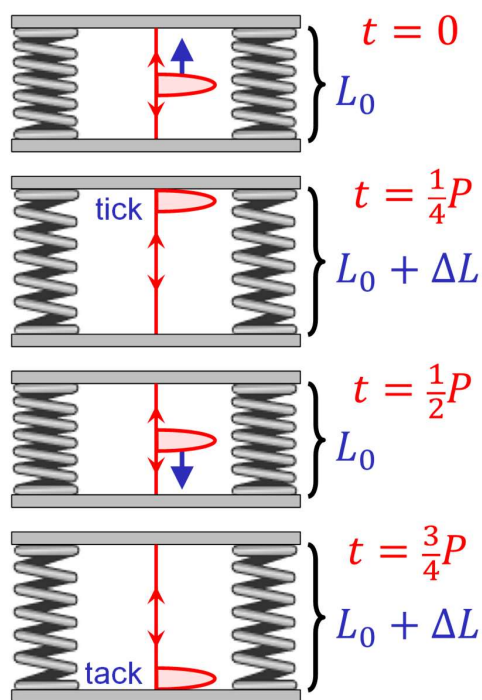


Figure 6. From top to bottom, four moments in time during one period P are shown. The mirrors of the cavity are held together by springs. Energy and momentum are exchanged between the light and the springs. In the situation shown, the cavity oscillates at twice the frequency of the clock.

As is clear from figure 6, momentum and energy is exchanged between the light and the springs. This means that the cavity is neither rigid nor massless. Even when springs and mirrors are massless to begin with, the stretched or compressed springs will acquire mass. This means that the light clock has become a system of coupled oscillators. What is nice about this picture is that it addresses all three issues for describing the basic nature of an elementary quantum particle as mentioned in the previous section, namely, the binding forces of the system are made explicit and the moving mirrors expresses the internal

oscillation to the outside world. The cavity can emit as well as reflect and absorb waves from the outside. For example, it has been shown [10] how a zero-point electromagnetic background may be modulated to represent the de Broglie wave around a particle. It is perhaps [11] unnecessary to mention that any physical light-speed wave should already be present in space before a beat can be propagated over it at the much faster-than-light phase velocity, in other words, the de Broglie wave does not exist outside the particle's light cone.

Also, it seems possible that the cavity oscillates at twice the frequency of the clock, similar to a transformer humming at 100 Hz as a result of the 50 Hz AC current running through it. However, whether this is truly the case does depend strongly on the mass, size and spring constant of the cavity compared to the light energy inside. Anyway, it seems to be justified to say that all three issues mentioned in the previous section may be addressed by replacing the rigid cavity by a more realistic cavity structure.

Some quantities describing the behavior of the system may be calculated to have some idea of the physics. The cavity will stretch and shrink due to a driving force

$$F_{rad} = \frac{\Delta p}{\Delta t} = \frac{2\hbar\omega}{L_0} = \frac{h\omega^2}{n\pi^2 c} \quad (26)$$

where $P = 2\Delta t = 2L_0/c = 2n\pi/\omega$, at least in case of a sturdy cavity with small ΔL . The force F and the stored energy E of the springs when deformed by length ΔL are

$$F = -\kappa\Delta L \quad (27)$$

$$E = \frac{1}{2}\kappa(\Delta L)^2 \quad (28)$$

If the cavity were rigid $\Delta L = 0$ and $\kappa = \infty$, and $E = -\frac{1}{2}F\Delta L = 0$ so that no extra energy would be stored in the springs when the light bounces up and down. In a realistic physical system, whenever $\kappa \neq 0$, it comes about from an internal binding energy (a zero point energy, in fact) that does represent a (rest)mass. Force balance between cavity and radiation, $F_{rad} = -F$ for a sturdy cavity gives

$$\kappa L_0 \Delta L = 2\hbar\omega \quad (29)$$

A soft cavity, with equipartition of energy between radiation and cavity, $E_{rad} = 2E$, gives

$$\kappa(\Delta L)^2 = \hbar\omega \quad (30)$$

The latter implies that $\Delta L \leq L_0/2$, and E_{rad} should be interpreted as the energy carried by the radiation when in a rigid cavity, or the total energy. In case of the soft cavity, on average, half the energy is in the springs, so that together they acquire a mass $m_{spring} = \hbar\omega/2c^2$. Now, with $\Delta L \approx L_0/2 = n\pi c/2\omega$, the dynamic spring constant as well as the mechanical resonance frequency of the cavity can be estimated:

$$\omega_{spring} = \sqrt{\frac{\kappa_{soft}}{m_{spring}}} \approx \frac{2\sqrt{2}}{n\pi} \omega \quad (31)$$

which means that the photon frequency and the frequency of cavity "breathing" are of the same magnitude when the cavity is single-mode. The estimated spring constant is:

$$\kappa_{soft} = \hbar\omega/(\Delta L)^2 \approx E_{rad}(4/n\lambda)^2 \quad (32)$$

A soft cavity will lead to strong frequency shift of the light when it is reflected from a mirror while simultaneously pushing it back. In turn this destroys the resonance condition and it is clear that the full behavior of a soft cavity coupled to the light inside provides us with a rich problem. From here, before getting carried away in silly calculations on what is likely going to be an inconsistent or unphysical construction of sub-atomic steel springs and silvered mirrors, it seems wise to ponder a bit on what the true nature of the cavity of a quantum particle might be.

4.2. *The cavity in the light of quantum mechanics*

Before all, we must face the fact that elementary particles do exist, and that they are somehow, however elusively, constructed according to a consistent set of rules from a kind of substance we do not clearly see, but nonetheless must be there. Nature as a whole, from the largest to the smallest, does operate in a consistent way and it cannot be so that there is total anarchy and arbitrariness at its foundations. This fact should not be blurred by, or confused with the notion that, indeed, in quantum mechanics we can only know the *outcome* of a measurement and from this measurement we cannot know the whole structure or detailed history that led to this outcome. Nonetheless, despite quantum uncertainty there is, and must be underlying consistency en logic. Indeed, this is the philosophy behind the de Broglie-Bohm theory of quantum mechanics also known as the realistic interpretation or pilot wave picture [12,13], and it is where the Copenhagen picture fails to inspire. The pilot wave's principal appeal is that it restores realism and determinism to quantum mechanics, its weakness that the physical nature of the guiding wave field has remained unclear.

In recent years, however, there have been some remarkable experimental and theoretical findings. A macroscopic pilot wave system has been discovered [14,15,16] that exhibits several features previously thought to be exclusive to quantum mechanics. A droplet (particle) bouncing on a liquid bath can self-propel due to its interaction with the waves (driving and guiding pilot wave) it generates [14].

In a way, to make complete the analogy between the macroscopic and microscopic world, what we propose in this paper is that the driving force should be not in the bath, but in the droplet. It is principally the cavity that is oscillating, not the bath, but their existence cannot seem to be separable or independent.

This brings us back to the question of what the nature of the cavity might be. The purpose of the cavity is to provide boundaries from which the wave can reflect and be localized. Usually these boundaries exist permanently in space, as for a rigid box. Because time and space play similar roles in wave propagation, manipulating time boundaries provides a complementary approach. For example, it has been shown [16] that sudden changes of the medium properties generate instant wave sources that emerge instantaneously from the entire space at the time disruption. As a matter of fact, in figure 6 we do have a time dependent boundary, the moving mirrors driven by the light inside the cavity. Drawing a parallel to the time-dependent medium mentioned above, the only more or less obvious substance available to create a cavity for the photon would be some dielectric structure as a result of a polarized vacuum. The dielectric and magnetic properties of this medium would have to oscillate with the light, and this implies that we have a so-called parametric oscillator. This would mean, in some way, that the light is waving in the space of the vacuum, and that the vacuum of the space is oscillating within the light.

A simple parametric oscillator is described by the following differential equation:

$$\frac{d^2\vec{E}}{dt^2} + \varepsilon(t)\frac{d\vec{E}}{dt} + \omega^2(t)\vec{E} = 0 \quad (33)$$

where \vec{E} is the field and $\varepsilon(t)$ the time-dependent dielectric constant. Proceeding along these lines the polarization fields of the vacuum are:

$$\vec{D} = \bar{\varepsilon}_r(t)\vec{E} \quad (34a)$$

$$\vec{B} = \bar{\mu}_r(t)\vec{H} \quad (34b)$$

and equation 33 should be replaced by the Maxwell equations, of course. The elements of the tensors $\bar{\varepsilon}_r(t)$ and $\bar{\mu}_r(t)$ are the parameters that are modulated by both of the fields \vec{E} and \vec{H} . Here it should be noted that time reversal as part of the coupled oscillation of medium and wave may require an electromagnetic negative refractive index [17]. All this of course begs the question of what the vacuum really is, but if our ideas may have any merit, they will certainly knit together the foundations: light (energy), particles and the vacuum of space-time.

In previous work it was envisaged [2,18] and argued [19] that the theory should allow topologically non-simple solutions, "knots" so to speak and it should support the description of spin- $\frac{1}{2}$ fields. To this

aim a combination of Maxwell electromagnetism, geometric algebra and Bateman's method [20] was developed [19]. To get to any solutions of a set of differential equations, the starting or boundary conditions are essential, and therefore it has also been the aim of this paper to get a better idea of the possible boundary structures one may be looking for to describe an elementary quantum(like) object in the proposed theory of reference [19].

5. Summary and conclusion

Starting with the concept of the light clock we have shown that a Lorentz boost to velocity \vec{v} on a single-photon standing wave produces a spatial beat pattern with exactly the de Broglie wavelength of a quantum particle of mass $m = \hbar\omega/c^2$. In the boosted frame there appears a beat wave vector \vec{k}_B which is related not to a single wave, but to a set of wave frequencies, the phases of which are in harmony at all the three different velocities v , c and v_{phase} . The de Broglie beat $\cos(k_B z - \omega_{phase} t)$ is a property of any mode of a light-speed wave inside any cavity of arbitrary shape. The beat propagates at phase velocity $v_{phas} = c^2/v$ and is internal to the clock cavity. Consequently, if the cavity is closed and rigid, the beat will be unobservable external to the cavity. However by allowing the cavity to be elastic, the binding forces being the equivalent of the Poincaré stresses that stabilize elementary particles, we find that the cavity and light wave form a coupled oscillator, As a consequence the cavity is itself oscillating, and this may be observable from outside of the clock. Then also the de Broglie beat may become observable both internal and external to the clock.

Speculation is dangerous and the main purpose of this paper has been to draw some parallel between the know behavior of quantum systems and that of the light clock. We have found that a cavity mode of a light-speed wave may be a basic part of the design rules of elementary particles. To drive the analogy further, it was suggested that the cavity may be elastic, or better it may be some dielectric structure as a result of vacuum polarization, parametrically coupled to a single photon that is thus localized.

In this paper it was shown how the de Broglie wavelength of a light clock and quantum particle may have the same origin, leading to the hypothesis that elementary particles may in essence be light clocks, the cavities of which are parametric oscillations of the vacuum polarization. Applied to earlier ideas, this hypothesis may guide us to appropriate boundary conditions that may help us find stable knotted field structures as solutions within a theory of topological electromagnetism.

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