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A new theory of light and matter

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Within a new mathematical framework, an extension to the free-space Maxwell equations is proposed with four further linear coupled differential equations. Within the formalism, a sharpening of the principle of relativity leads to new solutions of the Maxwell equations corresponding to uncharged rest-massless propagating states identified with the photon. The simplest extension, including a rest-mass density, allows for a self-confinement of the resulting system. The resulting new solutions are necessarily charged and essentially fermionic. These are identified with the electron and positron.

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1. Introduction

What is missing for the practical scientist or engineer in the present pantheon of physics is a fully relativistic theory of both electron and photon with which one can calculate at low energies. Here, an extension of the Maxwell equations, together with a sharpening of the principle of relativity, is proposed. Together, these lead to two new kinds of solutions: quantised photon-like wave-functions transforming correctly relativistically right down to the zero energy limit, and massive charged spin-half self-confined objects, corresponding to the spin-up and spin-down electron and positron. Perhaps because of the difficulty of quantising the theory in the usual way, there has been little prior art in extending the classical theory of fields during the last few decades, though a summary of work on this by Einstein and others has been provided by Waite[1]. The author and Dr. van der Mark proposed a semi-classical model of the electron as a localised photon[2] some time ago, which also contains some references of interest.

2. On the mathematical framework

This paper will use a relativistic algebra with four base-dimensions only. These are four unit “lines" in three spatial and one temporal dimension, transforming as the components of a 4-vector. Further, products and quotients of these are defined leading to the derivation of unit physical points, planes, volumes and a hypervolume. Under a general Lorentz rotation these transform as an invariant, a field, an angular momentum density and a dual invariant[3]. Primarily it is the strict constraints imposed by forcing the mathematics to conform to the physics (rather than the reverse) that leads to new results here.

Let a frame-independent unit four-vector basis in time and three spatial directions be denoted, α_0,α_1,α_2 and α_3 respectively. The product is defined such that the time element squares to the positive invariant scalar unit $\alpha_0^2 = +\alpha_P$, and the spatial elements square to the negative scalar unit $\alpha_1^2 = \alpha_2^2 = \alpha_3^2 = -\alpha_P$. These transform as 4-vector elements. The unit scalar in the algebra, α_P, is Lorentz invariant. In the description of a physical object, real numbers are used to represent a quantity, an extent, or a Lorentz scaling and a unit element α to represent the proper relativistic form. A convention is adopted where Greek indices run from 0 to 3 and Roman from 1 to 3.

A four-vector is written $\mathbf{v} = a_0\alpha_0 + a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = v_0 + \vec{v}$, where the a_μ are real number magnitudes. The over-arrow denotes a three-component object, here just the spatial part of the 4-vector. Using a summation convention, the square is written $\alpha_P(a_0^2 - a_1^2 - a_2^2 - a_3^2) = \alpha_P(a_0^2 - a_i^2) = a_\mu^2\alpha_P$. The unit element α_P squares to itself $\alpha_P^2 = +\alpha_P$, in the usual definition of idempotence. It is distinguished from the real or natural number 1 in that it is invariant under a Lorentz transformation. It may take only the two values $\pm\alpha_P$.

The operation of "addition" is not defined here for any of the unit elements, but is ascribed to the real number pre-factors representing an extent or a magnitude. The sub-algebra of multiplication of the α unit elements amongst themselves is isomorphic to the Clifford algebra Cl(1,3). It is also related to Dirac algebras. Loosening the restrictions and adopting a complex, rather than a real pre-factor would extend to the algebra to encompass Dirac algebras.

The ordered product or quotient of one spatial unit element with another, for example $\alpha_1\alpha_2$ gives a unit right-handed ordered spatial plane (bivector) element. This spatial plane is denoted

$\alpha_1\alpha_2 = \alpha_{12}$. The reverse ordering gives a plane in the opposite (left-handed) direction, that is $\alpha_{12} = -\alpha_{21}$. There are three such right-handed objects: $\alpha_{12}, \alpha_{23}, \alpha_{31}$. Products such as $\alpha_1\alpha_0 = \alpha_{10}$ represent planes in space-time. Together, these six (bivector) elements inherit the transformation properties of fields[3]. The former transform as the magnetic and the latter as the electric field. Further, there are 4 tri-vectors (α_{123}, α_{023}, α_{031}, α_{012}). The latter three transform as as an angular momentum density. Finally, there is a pseudoscalar quadrivector (α_{0123}) which is, just as the scalar, invariant under a general Lorentz transformation but squares to negative scalar unity $\alpha_{0123}^2 = -\alpha_P$.

Now physics is introduced. The mathematics is forced to follow the physics by imposing a constraint, called here the principle of absolute relativity, such that all quantities *must* appear together with their proper relativistic form. In particular, spatial and temporal intervals are written $\alpha_i\Delta x_i$ and $\alpha_0\Delta x_0$ respectively. This may appear at first sight quite uncontroversial, but this principle will be imposed at all levels, including in differentials, constant factors and the phase-factors of exponentials. This ensures that elements derived from any expansion or phase development transform correctly with respect to one another under general Lorentz transformations.

For Cartesian co-ordinates a 4-vector 4-differential is written:

$$d = \frac{\partial}{\alpha_\mu \partial x_\mu} = \partial_\mu/\alpha_\mu = \alpha_0\partial_0 - \alpha_1\partial_1 - \alpha_2\partial_2 - \alpha_3\partial_3 = \alpha_0\partial_0 - \alpha_i\vec{\nabla} \tag{2.1}$$

Note the change of sign of the space components due to the implicit quotient of the unit vectors squaring to negative unity. A 4-vector potential in some frame may be written :

$$A = \alpha_0 A_0 + \alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3 = \alpha_0 A_0 + \alpha_i\vec{A} \tag{2.2}$$

and taking the 4-derivative dA in this frame yields 16 $(= 1+3+3\cdot 2+3\cdot 2)$ terms. Writing these terms out in full, it is immediately apparent that the patterns correspond precisely to those in the 3-space notation of div, grad and curl. Explicitly:

$$dA = \alpha_P(\partial_0 A_0 \, + \, \vec{\nabla}\cdot\vec{A}) - \alpha_{i0}(\partial_0\vec{A} + \vec{\nabla}A_0) - \alpha_{ij}\vec{\nabla}\times\vec{A} = P + F \tag{2.3}$$

which is the sum of a scalar (pivot) part P and a bivector (field) part F. In Eq. (2.3) the term in α_{i0} is usually identified[4] with the electric field $\vec{E} = -\partial_0\vec{A} - \vec{\nabla}A_0$ and that in α_{ij} with the magnetic field $\vec{B} = \vec{\nabla}\times\vec{A}$.

The general case is considered. To encompass the more general form of the Maxwell equations $dF = J$ including a charge and current density, a distinction is drawn between two types of quantity. The first is a dynamical quantity, governed by the equations to be derived. For these, the operation of the 4-vector derivative, equation (2.1), whatever underlying physical process that may represent, will transform the quantities operated upon either down a grade (indices equal), or up a grade (indices differ).

The second is an eventual set of quantities which may be static, put in by hand, or arising from some external process such as an interaction, which are not considered as arising from a local 4-vector differential. This is, for example, the way in which charge and current are added in the simplest generalisation of the Maxwell equations.

Over each of the sixteen multivector-quantities defined above, a general dynamical multivector field G is defined over a scalar term $P\alpha_P$, a vector potential $A_\mu\alpha_\mu$, a field term $F\alpha_{\mu\nu} =$

$E_i\alpha_{i0} - B_i\alpha_{jk}$, a trivector potential term $T\alpha_{\mu\nu\rho}$ and an eventual quadrivector potential $Q\alpha_{0123}$ such that: $G = P\alpha_P + A_0\alpha_0 + A_i\alpha_i + E_i\alpha_{i0} - B_i\alpha_{jk} + T_k\alpha_{0ij} + T_0\alpha_{123} + Q\alpha_{0123}$. In an obvious notation, the external terms are defined as $C = C_P\alpha_P + C_0\alpha_0 + C_i\alpha_i + C_{i0}\alpha_{i0} + C_{jk}\alpha_{jk} + C_{0ij}\alpha_{0ij} + C_{123}\alpha_{123} + C_Q\alpha_{0123}$.

Writing, by analogy with the form of the Maxwell equation $dF = J$, $dG = C$, and again using the conventional 3-space patterns for reference, one obtains from the odd terms a generalisation of the Maxwell equations as:

$$\alpha_0(\vec{\nabla}\cdot\vec{E} + \partial_0 P) = C_0\alpha_0 \tag{2.4}$$
$$\alpha_{123}(\vec{\nabla}\cdot\vec{B} + \partial_0 Q) = C_{123}\alpha_{123} \tag{2.5}$$
$$\alpha_i\left(\vec{\nabla}\times\vec{B} - \partial_0\vec{E} - \vec{\nabla}P\right) = C_i\alpha_i \tag{2.6}$$
$$\alpha_{0ij}(\vec{\nabla}\times\vec{E} + \partial_0\vec{B} + \vec{\nabla}Q) = C_{0ij}\alpha_{0ij} \tag{2.7}$$

and four new equations in the even terms:

$$\alpha_P(\vec{\nabla}\cdot\vec{A} + \partial_0 A_0) = C_P\alpha_P \tag{2.8}$$
$$\alpha_{0123}(\vec{\nabla}\cdot\vec{T} + \partial_0 T_0) = C_Q\alpha_{0123} \tag{2.9}$$
$$\alpha_{i0}\left(\partial_0\vec{A} + \vec{\nabla}A_0 + \vec{\nabla}\times\vec{T}\right) = C_{i0}\alpha_{i0} \tag{2.10}$$
$$\alpha_{jk}\left(\partial_0\vec{T} + \vec{\nabla}T_0 - \vec{\nabla}\times\vec{A}\right) = C_{jk}\alpha_{jk} \tag{2.11}$$

In the first four, the usual free-space Maxwell equations are recovered by setting P and Q to zero. In these, setting C_0 to the charge, C_i to the current density and the other constants to zero one obtains exactly the usual inhomogenous Maxwell equations.

With $C_P = 0$, equation (2.8) is just the Lorenz gauge condition. C_P non-zero expresses the usual gauge degree of freedom. For the case that T is zero eqs. equation (2.10) and equation (2.11) are just the usual relations between the 4-vector potential and the electric and magnetic fields respectively.

For the simple case of a propagating free-space electromagnetic wave, and forcing z and t to take the proper form α_3 and α_0 respectively according to the principle of absolute relativity, a solution of the field distribution of a left circularly polarised electromagnetic wave travelling in the the $+z$-direction for the free space case may be written:

$$F_L = RH_0U\omega(\alpha_{10} + \alpha_{31})e^{R(\omega\alpha_3\frac{z}{c} - \omega\alpha_0 t)\alpha_{012}} \tag{2.12}$$

Where the real-number constant c is the (scalar) speed of light. H_0 is an arbitary (real) distribution representing the field spread in phase. Its square integrates to unity. R is a frame-matching factor and is unity in the centre of mass frame and for emitter and absorber in the case that both are in the same frame. U is a universal factor, the same for all photons, with proper dimensions to convert the whole expression to field quantities. The angular frequency $\omega = \mathscr{E}/\hbar$ must appear in three places for this to be a pure-field solution with the correct Lorentz transformation properties, twice in the exponent and once in the pre-factor. The introduction of the form α_{012} corresponds to a unit angular momentum. This corresponds, clearly, to $\hbar$. Without this proper form the solution is not propagating. The exponential is not, by itself, a solution to the Maxwell equation $dF = 0$ but is a solution to the more general equation $dG = 0$. The whole expression becomes a field only solution to the first-order equation $dF = 0$ if a pre-factor corresponding to the experimentally observed configuration of equal and perpendicular electric and magnetic fields is introduced. In this case equation (2.12) represents the field pattern of a left-circularly polarised electromagnetic field [3]. It differs from conventional solutions in that the stringent restrictions on its form imposed by the condition of “absolute relativity" forces light to come in "lumps": photons.

Under a Lorentz transformation the integrated energy varies directly with the frequency, leading to a relation $\mathscr{E} = h\nu$, where h is some universal constant. Interference means that solutions may not be added at the same point in space-time, so the only way to change the energy transmitted in an event is to change the frequency. For an estimate of the value of $\hbar$ in terms of the elementary charge q or vice-versa the reader is referred to earlier work[2].

Though this new photon-like solution is an important result in itself, the first of this paper, it will not be discussed further here since the main theme is to investigate how the introduction of a dynamical rest-mass term, $P\alpha_P$, may modify its behaviour to turn the twisting electromagnetic field of equation (2.12) into a topologically non trivial configuration with half-integral spin and with quantised charge. Consider now the simplest possible extension, where all the constant terms are zero and one introduces only a non-zero dynamical pivot term $P\alpha_P$ into the extended Maxwell equations. That is the equation $d(F+P) = 0$. It is not claimed here that this is necessarily the complete new equation governing the existence of and the full internal motion of the electron, that should involve more terms in the general equation. It does, however, introduce one new, essential feature into the theory of electromagnetism: a term allowing electro-pivot-magnetism to confine itself.

Consider the energy-momentum density for two counter-propagating fields. This corresponds to the situation at the point of electron-positron pair creation. Denoting the 7-component field and pivot as $G = F + P$ and the conjugate set as $G^\dagger = F^\dagger + P^\dagger$ their product gives:

$$M_{field} = \frac{1}{2}(F+P)(F^\dagger + P^\dagger) =$$
$$\frac{1}{2}\alpha_P(\vec{E}^2 + \vec{B}^2 + P^2) + \alpha_{i0}(\vec{E}\times\vec{B} + P\vec{E}) \tag{2.13}$$

It is immediately apparent that, for the case $P = 0$, one obtains the usual expression for the electromagnetic energy density $\frac{1}{2}(\vec{E}^2 + \vec{B}^2)$ and the momentum density (the Poynting vector) $\vec{E}\times\vec{B}$ as expected. The new feature for $P \neq 0$ is the emergence of an extra term in the rest mass-energy density (P^2) and an extra term in the momentum density ($P\vec{E}$). The latter term turns the direction of momentum propagation in the direction of the electric field, allowing the possibility of closed flows corresponding to physical particles. Within the extended theory, force-free motion ($d(GG^\dagger) = GdG = 0$) requires the pivot to be confined within existing particles.

It has long been known in the field of mesoscopic physics that one can study the “charge" distribution within single electrons[5], though one is really investigating the collective behaviour of a set of interacting “elementary" particles. The underlying motivation for this work is to develop a formalism to properly understand the inner workings of such systems beyond the primitive insights that may be gleaned from non-relativistic quantum mechanics.

The configuration required for a free particle may readily be modelled in the space of phase and momentum such as that presented in equation (2.12). Modelling the twist in this space as a torsioned strip and turning a single full twist to turn to meet itself after a single wavelength results in the model presented in earlier work[2] and illustrated in Fig. 1. This is equation (2.12) in toroidal form and with the addition of a pivot pre-factor. The figure is not drawn in space, but in the (bivector) space of $\alpha_{12}, \alpha_{23}, \alpha_{31}, \alpha_{10}, \alpha_{20}$ and α_{30}. Though the figure is toroidal in momentum space, the projection onto ordinary space is spherical. Experimentally, as is well known, the electron is spherically symmetric down to length scales smaller than the limit of the classical electron radius. A slice through either space (perpendicular to the local momentum direction) yields a circular section with orthogonal electric and magnetic fields. In the internal toroidal space the photon propagates in the axial direction ϕ, rather than as in equation (2.12) where the propagation is in z. The momentum-flow of the internal electro-magnetic wave is now constrained by the pivot term P to circulate in the toroidal bivector space. To be simultaneously a solution to equation (2.13), this must be in a direction everywhere perpendicular to the (radial) electric field. The resulting electric field, if it could be observed in normal space by a (mythical) all-seeing quantum observer, would appear to rotate on a path similar to the seam of a (vanishingly) small tennis-ball about the centre of momentum, remaining radial. As a result of confinement, such flows appear “charged" to an external observer.

An interpretation of the internal structure as a simple torus in space[1] (as opposed to momentum and phase space), is certainly a model of the electron, but since there is no simple point centre of mass about which such a system would rotate, not quite just the same one as is envisioned here. Note, however, that the present model implies that the effective centre of spin and the centre of charge should differ, as has recently been observed in experiment[6, 7].

This model embodies the quantum particle. What is waving, just as in the case of the photon, is the field, but now in harmony with a dynamical pivot term P. What it is waving in is the confinement engendered by the interaction of the radial electric field with the new, dynamical, rest-mass term P.

It has been stated that “charge" does not really exist within the new theory and yet that the configuration is charged due to the confinement. This comes to one of the central, outstanding mysteries of physics. What is the underlying nature of quantised charge?

For the solution envisaged above, both the existence of a radial electric field and mass-energy exchange are essential features, though the electron is no longer viewed as a point particle as in quantum electrodynamics but has some complex internal spherically symmetric structure. For non-zero pivot, the electric field divergence in equation (2.4) is no longer zero but is but is related to the rate of mass-energy exchange between existing particles $\alpha_0 \partial_0 P$. In the new theory, there is no underlying charge density. Charge manifests only as a result of the reconfiguration of the field by the confinement topology to be everywhere radial. It appears only in the self-confined field-particles themselves. It is created only in the process of fermion-antifermion pair creation. It is the detailed conservation of the electromagnetic fluid flow, especially the conservation of angular momentum (vorticity), that requires charge conservation. The model then leads to a relation between the quantised angular momentum and the quantised charge, as was calculated in earlier work[2].

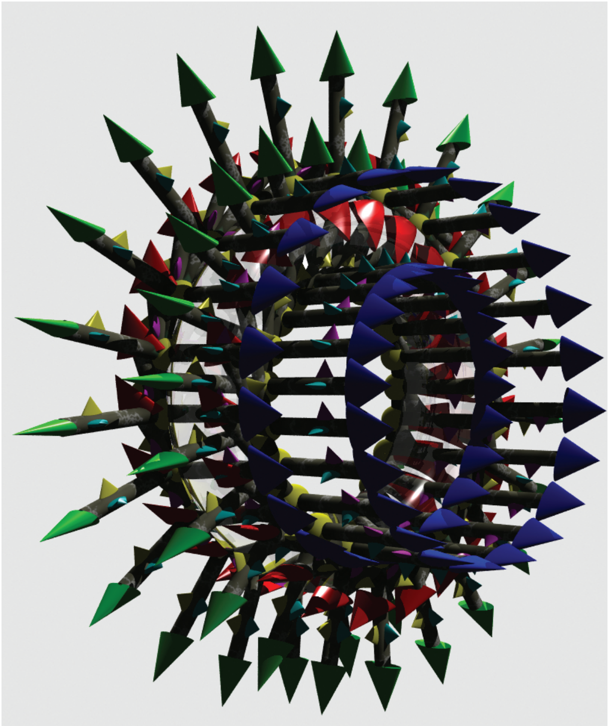

Figure 1: Field and momentum space snapshot for the positron. The electric field vector direction is represented using green, the magnetic field direction using blue and the momentum density red arrows. Circular sections through the torus correspond to spheres in normal space.

What is exchanged between existing particles is mass-energy (pivot), intermediated by (necessarily quantised) photons of the form of equation (2.12). An electron with more pivot winds tighter, oscillates at a higher frequency and contains more energy. Pivot and field increase as the inverse square of the field, giving a fourth-power increase in energy density. The volume reduces with the third power leading to, just as is the case for the photon solution of equation (2.12), an energy increase linear with frequency.

From the apparent radial external field for a free particle it is possible to estimate a value for the elementary charge. A lower limit may be obtained for an infinitesimally thin ring, corresponding to a minor axis for the torus of zero radius. Then both loops lie on top of each other at the characteristic object size of $r = \lambda_c/4\pi$. A radial field with this effective radius corresponds to an effective charge of $q = \pm\frac{1}{2\pi}\sqrt{3\varepsilon_0\hbar c} \approx 0.91e$, as has been shown in earlier work[2]. Note that this calculation is size and mass scale independent, giving the same charge for other elementary particles of different mass such as the muon and tauon. The charge is due to the confinement topology, not the size. The apparent charge increases as the minor radius is increased with respect to the major radius up to a limit of a few tens of the electron charge. The minor radius is then constrained by the physically observed charge, and by high energy scattering experiments, to be quite close to the limiting path where it is small with respect to the major radius.

Given the momentum magnitude p of the constituent photon, it is straightforwards to calculate the spin of the double looped object $\mathbf{r} \times \mathbf{p}$ and it is found to be half-integral[2]. By the spinstatistics theorem, therefore, the proposed object is a fermion. More fundamentally, as is manifest from the figure, the object must rotate by 720 degrees to return to its starting position. This is a key property of a fermion and the model constitutes a physical spinor. Further, as has been argued elsewhere[8], a consideration of the overlap of the internal fields leads to an excess energy of the order of the particle mass in the spin parallel case, and a smaller or zero increase in energy for the spin antiparallel case. This gives a possible physical origin for the exclusion principle itself. That the exclusion principle corresponds to an energy of this order has been clear experimentally for some time[9].

3. Conclusions

A new set of coupled, linear differential equations, encompassing the homogenous Maxwell equations has been proposed. For light, the solutions require a quantised angular momentum and their energy scales with frequency. The quantisation is not imposed mathematically, but is required physically for the solution to be propagating. The introduction of a new scalar term leads to circulating electro-pivot-magnetic vortices in momentum space. Such configurations are necessarily charged and have half-integral spin. They are identified with the electron and positron.

Though the results of this simple extension of the Maxwell equation have some interesting features, it is the view of the author that this is not the whole story. In particular, it is likely that the trivector and quadrivector terms should also play a role in constraining allowed solutions.

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