FERMIONS FROM BOSONS AND THE ORIGIN OF THE EXCLUSION PRINCIPLE

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Abstract: *Based on a model of the electron as a localised photon, a possible physical basis for the Pauli exclusion principle is proposed. It arises from the harmonic interference of the underlying fields. One possible theory leading to the self-confinement of electromagnetism is explored: an extension of the Maxwell equations.*

Keywords: *Electron, electromagnetism, exclusion principle, electromagnetic vortex*

1 Introduction

The experimental fact that gauge vector bosons may be created from fermions in the process, for example, of electron-positron annihilation is the starting point for this paper. On the face of it, this suggests that either the photon may be described in terms of elementary particles, or that elementary particles may be described in terms of photons, or that both may be derived from a deeper principle. Based on this premise, many authors have attempted to develop models of particles as a pure, localised field distributions. A review of some of this work may be found in a paper by Waite[1]. He also develops on a study by Einstein and derives a toroidal model of the electron as localised electromagnetism.More recently other authors have developed similar models, based on somewhat different sets of initial premises[2, 3, 4, 5].

In previous work we have outlined a semi-classical theory of the electron as a localised photon[4]. The paper differed from the approach of Waite in that the space in which the photon was confined was a combination of space and time (corresponding to momentum space) rather than normal space. Briefly, the paper showed that, by considering the electron as a localised photon, either the elementary charge or the half-integral spin could be derived from the other. It also showed a physical basis for the anomalous magnetic moment of the electron and the reason that the model electron, an object extended at the length scale of the Compton wavelength, could appear point-like in high energy physics scattering experiments. The model made contact with the zitterbewegung motion of the Dirac electron in that it had an intrinsic frequency of twice the electron Compton frequency[7, 9, 10, 8], a feature which is now being probed in recent experiments[11].

The simplest configuration of a single-wavelength photon making a single complete turn of the fields about the momentum direction has a double turn in the space of momentum, where the streamlines of momentum flow are over a family of nested torii. How the field distribution for a circularly polarised photon, where the fields rotate about the propagation direction to produce a helical pattern, is itself rotated to form a configuration with charge and half-integral spin is described in some detail in the earlier work[4]. Briefly, the field rotation (the twist about the propagation (momentum) direction, is rotated into the same plane as the turn of the momentum flow (given by the Poynting vector), resulting in a momentum flow about a family of nested torii. For this flow the electric field is everywhere outward directed, leading to a charge.

A configuration of this form is illustrated in Fig. 1. The vortex corresponding to the positron has been chosen, since outward-directed electric fields are easier to draw.

The main aim of this paper is to argue that the physical cause of the Pauli exclusion principle itself is that of a very strong force, as strong or stronger than the strong interaction, arising from the interference of the internal field configuration of the fermions. That such forces exist was first apparent in polarised proton scattering[12, 13].

Given the form proposed in earlier work and illustrated in the figure, it is quite straightforwards to understand the proposed physical origin of the exclusion principle and this is handled quickly. The main body aims to address one weakness of the earlier work: that the photon confinement mechanism was merely postulated. A development of the Maxwell theory of electromagnetism is proposed. Rather than introducing a raft of properties for the electron such as charge, spin and mass, a simple term is introduced leading to all of them. The term also provides a mechanism for the self-confinement of electromagnetism. The subsequent discussion serves to throw light on the nature of the space in which the photon is confined, lays bare precisely its relationship to normal space and time and allows a discussion of the uncertainty principle and wave-particle duality.

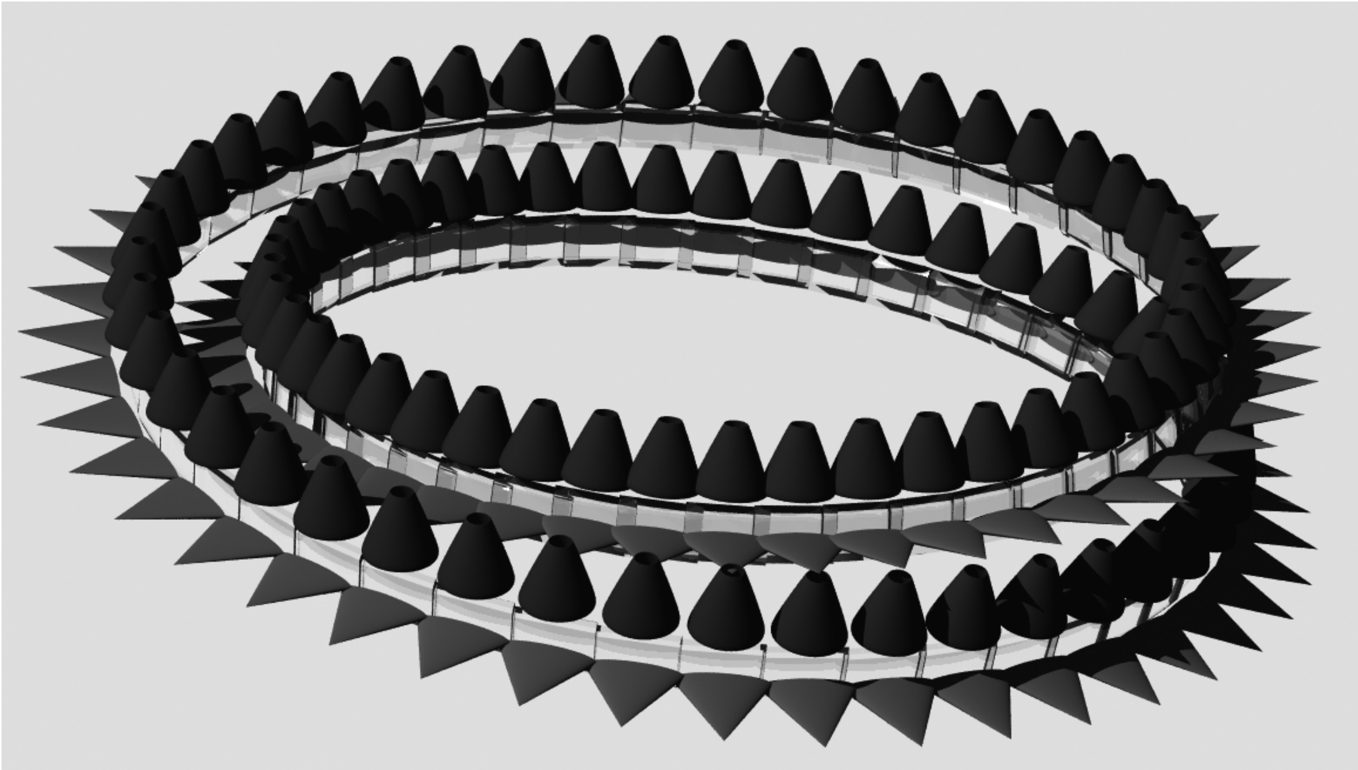

Figure 1: Field and momentum snapshot for a spin-up positron. The electric field vector direction is represented using light gray arrows, the magnetic field direction using dark gray hollow cones. The momentum path is represented in glass. A full mode structure will fill space with tumbling toroidal shells and be spherically symmetric.

2 Charged fermions from uncharged bosons, the exclusion principle and doubly charged bosons from charged fermions

Starting from the field configuration shown in Fig. 1, it is possible to see that such electromagnetic vortices are fermions at three levels.

At the level of spin, at the level of symmetry and, most importantly, at the level of the exclusion principle itself.

Given the spin of the constituent photon, it is straightforward to calculate the spin of the double looped object[4] and it is found to be half-integral. Therefore, by the spin-statistics theorem, the proposed object is a fermion.

More fundamentally, as is clear from Fig. 1, the object is double-covering over the torus in momentum space and returns to its starting configuration after a 720 degree rotation in the space of an outside observer. That is the object has the intrinsic symmetry of a fermion.

The above arguments, however, are only of value within some theoretical framework which supports them. The compelling experimental manifestation of fermionic behaviour is the Pauli exclusion principle. Any advance in understanding needs to show why there should be little or no energy cost to impose two identical fermions with opposite spin while there is an enormous energy cost to impose two identical objects of the same spin on one another. That is the main aim of this paper.

Consider the overlap of the field components within the space of Fig. 1. If two such identical field configurations are superimposed the fields add, leading to a doubling everywhere of field intensity.This is in contrast to the case for photons in free space, where constructive interference may occur at one place(or time) but destructive interference will compensate at another, leading to the possibility of adding such configurations linearly, a bosonic behaviour. Here, since the mass density increases as the square of the field, superposition leads to a fourfold increase in the mass. That is, the total mass energy of two superimposed spin-up positrons would require a four electron mass-worth of energy. This leads to a force, resulting from the constructive interference of the internal field components, which is stronger even than the strong force. It is proposed that it is this force that is responsible for the exclusion principle. Put simply, imposing two electrons or two positrons into the same state requires a doubling of the mass. The investigation of whether the magnitude is correct here should be accessible to experiment.

What of the case where a spin-up electron is superimposed on a spin-down electron such as in the inner shell of a Helium atom for example? Here, the radially directed electric field component is, as before, doubled but, as is clear from Fig. 1, the axial magnetic field component cancels. The electric field component doubles, the charge doubles, but the magnetic field component, reversed in the spin-down case, cancels. This leads, on performing the square and integration, to no net energy required for the field components. This is proposed as the underlying physical reason why opposite spin fermions may be superimposed. The underlying physics is that of the constructive interference of harmonic fields. This forms the primary result of this paper.

3 Towards a complete theory of light and matter

A new theory of electromagnetism is proposed. The theory includes a possible mechanism for the confinement of the internal photon, lays bare the space in which this confinement takes place and outlines a field theoretic basis for the earlier semi-classical model.

The development requires a new kind of physically-based mathematics. The algebra introduced is defined by the way the physical quantities it represents transform under Lorentz transformations and under rotations. It is designed to parallel as closely as possible the properties of points, lines, areas, volumes and hypervolumes in special relativity. In particular it requires at least two distinct objects, both behaving like the real number 1. One of these is used to denote the scale or number of a quantity and will be denoted γ_S. This has the properties of the ordinary real number 1 and will be omitted where this causes no confusion. The second corresponds in some ways to a point, as distinct to a line or a plane, but it is a point which has particular transformation properties associated with it. To distinguish it, the new unit element will be denoted γ_P. The subscript here refers partly to its physical properties in that it transforms as a rest mass (a ponderous mass), partly that in electromagnetism it turns out to act, in conjunction with the fields, as a pivot about which the fields turn leading to the self-confinement of the fields and partly that it represents a physical point (a point, not in size but rather as opposed to a line or a plane or a volume) in the algebra to be developed. Its main virtue is that it may be used to store energy at a point in space where, for example, fields cancel. The differential of the new quantity is related to a kind of gauge, though, as introduced it acts as a dynamical rather than a static gauge.

A gamma notation is used because the closest familiar mathematics in current use is one of the Dirac gamma matrix algebras. A further description, differing marginally from that used here, may be found in a recent thesis[6].The mathematics is based on merely the usual four dimensions of space and time, but leads to further distinct unit elements under multiplication and division. A unit time vector is written as γ_0 and transforms as the time component of a four-vector. It squares to the positive invariant scalar unity γ_P. Three orthonormal unit space-like vectors, squaring to negative γ_P are denoted as γ_1,γ_2 and γ_3 respectively. These transform as the spatial components of a four-vector. A distinction is made between division on the one hand e.g. $\gamma_1/\gamma_1 = \gamma_S$ and multiplication on the other e.g. $\gamma_0\gamma_0 = \gamma_P$. Magnitudes are represented by a product, for example a line of length three units in the 2 direction is $3\gamma_2$ and a point of magnitude 42 is $42\gamma_P$. Though this set of choices leads to a proper derivation of the Maxwell equations, it is not the only one which does so. The choice of which set of these best parallels the considerable complexity which arises under products and quotients must be derived from a series of experiments. The algebra is independent of the co-ordinate system and the equations to be derived are valid in any well-behaved conformal orthonormal co-ordinate system.

The ordered product or quotient of one spatial unit element with another, for example $\gamma_1\gamma_2$ leads to a unit right-handed ordered spatial plane (bivector) element. This spatial plane is denoted $\gamma_1\gamma_2 = \gamma_{12}$. The reverse ordering gives a plane in the opposite (left-handed) direction, that is $\gamma_{12} = -\gamma_{21}$. There are three such right-handed objects: $\gamma_{12}, \gamma_{23}, \gamma_{31}$. Note that these elements are linearly independent of each other and of the base elements. They are derived from the base dimensions under multiplication or division.Products such as $\gamma_1\gamma_0 = \gamma_{10}$ represent planes in space-time. These elements transform as fields.

Further, there are several choices to be made about the handedness and ordering of the operations between the various unit elements. In particular, the time element may be taken to come first or last (implying a change of sign). Both choices give a right-handed set of products amongst each other ($\gamma_1\gamma_0 \times \gamma_2\gamma_0 = \gamma_0\gamma_1 \times \gamma_0\gamma_1 = \gamma_1\gamma_2$. The convention taken is that which works with the standard left-to-right ordering of products and the standard handedness of co-ordinate systems. With the convention that the base elements $\gamma_1, \gamma_2, \gamma_3$ are right-handed, this ordering, with space first then time, forms a right-handed set for angular-momentum products (such as $r \times p$), the reverse ordering a left-handed one. The conventional signs in the Maxwell equations then arise if one adopts the convention that the multiplication of a unit vector in the 1 direction into an inverse unit vector in the 2 direction has the reverse sign to the simple product. That is $\gamma_1/\gamma_2 = -\gamma_{12}$. Considering all products, there are twelve linearly independent objects derived under multiplication: the Lorentz scalar (γ_P) invariant under a Lorentz transformation, six bi-vectors (γ_{10},γ_{20},γ_{30},γ_{12},γ_{23},γ_{31}) which transform as fields, 4 tri-vectors (γ_{123},γ_{012}, γ_{023}, γ_{031})which transform as angular momenta and a quadrivector (γ_{0123}) which is invariant under a Lorentz transformation but may change sign under other operations such as Hermitian conjugation[6]. These are derived unit points, planes, volumes and hypervolumes formed from the unit lines, γ_0,γ_1,γ_2 and γ_3 of the base vector set (which transform as a 4-vector). Taking γ_P as a base dimension there are 4 base and 12 derived dimensions, giving 16 linearly independent unit elements in all (or 17 if γ_S is included ,though this is not customary). The system so defined reduces to one of the Dirac algebras and to one of the Clifford algebras under the simplification of setting γ_S=γ_P. In practice the extra provisos of ordering discussed above are also important. In the following, the proper form of quantities in one of these directions will be represented by a token with ordered lettering, thus $\gamma_{\mu\nu}$ represents a bivector and, γ_{0ij},γ_{ij} and γ_{i0} are right-handed trivectors, space-space bi-vectors and space-time bi-vectors respectively. Here and in the sequel, Greek indices run from 0 to 3 and Roman from 1 to 3.

For Cartesian co-ordinates a 4-vector 4-differential is defined as:

$$\begin{aligned} d &= \frac{\partial}{\gamma_\mu \partial x_\mu} = \partial_\mu / \gamma_\mu \\ &= \gamma_0\partial_0 - \gamma_1\partial_1 - \gamma_2\partial_2 - \gamma_3\partial_3 = \gamma_0\partial_0 - \gamma_i\vec{\nabla} \end{aligned} \tag{1}$$

The over-arrow denotes a conventional three-vector. Note the change of sign of the space components due to the implicit quotient of the unit vectors. This behaviour turns out to make redundant the need to distinguish co- and contra-variant quantities. If one wishes, these may be considered to be related to the product and quotient, though this is not quite precise as the implicit ordering also may play a role. For further discussion of these points see a recent thesis[6]. The 4-differential of a 4-vector potential yields field components. Writing the vector potential as:

$$A = \gamma_\mu A_\mu = \gamma_0 A_0 + \gamma_1 A_1 + \gamma_2 A_2 + \gamma_3 A_3 = \gamma_0 A_0 + \gamma_i \vec{A} \tag{2}$$

The 16 ($= 1 + 3 + 3 \cdot 2 + 3 \cdot 2$) terms of the 4-derivative of the 4-potential dA may be gathered together and written as:

$$dA = \gamma_S(\partial_0 A_0 \quad + \quad \vec{\nabla} \cdot \vec{A}) - \gamma_{i0}(\partial_0\vec{A} + \vec{\nabla}A_0) - \gamma_{ij}\vec{\nabla} \times \vec{A} \tag{3}$$

which is the sum of a scalar part L and a bivector (field) part F.

In Eq. (3) the term in γ_{i0} is identified with the electric field $\vec{E} = -\partial_0\vec{A} - \vec{\nabla}A_0$ and that in γ_{ij} with the magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$. Together these terms form a bivector F which corresponds to the antisymmetric Faraday or field-strength tensor $F^{\mu\nu}$ [14].

$$F \quad = \quad \gamma_{i0}\vec{E} - \gamma_{ij}\vec{B} \tag{4}$$

The electric field maps to the set of three ordered space-time unit elements $\gamma_{10}, \gamma_{20}, \gamma_{30}$.

Writing the (six-component) electromagnetic field as F, all four free-space Maxwell equations may be written in the new formalism as $dF = 0$[6]. That is, the 24 terms of this product fall into two groups of three and two singlets (eight linearly independent components in all), giving exactly the four Maxwell equations. No dual field or antisymmetric element[14] is required as these properties are taken care of by the potency of the present formalism. Including a possible scalar term for $d(F + P) = 0$, multiplying out the terms and gathering the terms in the familiar 3-space quantities one obtains explictly:

$$\gamma_0\vec{\nabla} \cdot \vec{E} = -\gamma_0\partial_0 P \tag{5}$$

$$\gamma_{123}\vec{\nabla} \cdot \vec{B} = 0 \tag{6}$$

$$\gamma_i\left(\vec{\nabla} \times \vec{B} - \partial_0\vec{E}\right) = \gamma_i\vec{\nabla}P \tag{7}$$

$$-\gamma_{0ij}(\vec{\nabla} \times \vec{E} + \partial_0\vec{B}) = 0 \tag{8}$$

Setting the scalar P to zero gives exactly the full set of free-space Maxwell equations with all the correct signs. At this point, one could simply introduce a charge and current density vector on the right yielding the complete set of Maxwell equations. This will not be done a-priori here, as the preference is to derive charge rather than insert it. It is worth noting that an ultimately unsuccessful attempt was made by Dirac to bring charge in as a gauge term [15, 16]. In that work the extra term was brought into the Lagrangean and through to the Hamiltonian in the usual way. Here the new term is added directly to the equations of motion.

While this may be a compact way of writing these equations, this constitutes, in many respects, but little practical progress over the situation in the nineteenth century. To make progress, we move to a discussion of the energy and momentum density in the fields and the internal forces in the process of pair creation within a purely electromagnetic particle, where the new term introduced on the right-hand side above is neither zero nor constant. The energy-momentum density in the fields is obtained by multiplying the field by its conjugate. Denoting the 7-component field and pivot as $F + P$ and the conjugate set as $F^\dagger + P^\dagger$ in the product gives:

$$M_{field} = \frac{1}{2}(F + P)(F^\dagger + P^\dagger) =$$
$$\frac{1}{2}\gamma_P(\vec{E}^2 + \vec{B}^2 + P^2) + \gamma_{i0}(\vec{E} \times \vec{B} + P\vec{E}) \tag{9}$$

One sees immediately that, for the case $P = 0$, one obtains the usual expression for the electromagnetic energy density $\frac{1}{2}(\vec{E}^2 + \vec{B}^2)$ and the momentum density (the Poynting vector) $\vec{E} \times \vec{B}$ as expected. The new feature for $P \neq 0$ (corresponding to an invariant mass term) is the emergence of an extra term in the energy density (P^2) and an extra term in the momentum density ($P\vec{E}$).

From the equation, the effect of the pivot term on the electric field component is to introduce a momentum component perpendicular to the Poynting vector, leading to the possibility of a circulating purely electromagnetic wave. It is clear that for a sufficiently strong pivot contribution, a simultaneous solution to Eq. (9) and the Maxwell equation is consistent with Fig. 1. The development of the fields is similar to that in the photons (consisting of a twist about the momentum direction) but the momentum component is itself modified by a component parallel to the electric field direction, leading to a turn. A solution consistent with Eq. (9) is then a combination of a twist of the fields in a plane transverse to this resultant momentum direction but now combined with a turn of the momentum vector introduced by the $P\vec{E}$ term. For a solution the two rotations must be combined in such a way that the electric field is everywhere outward- (or inward-) directed. Such solutions are necessarily charged. Charge conservation is ensured in that such objects must be created in pairs.

Note, however, that the circulation is not expressed in normal space, but in the space derived under the quotient operation corresponding to energy-momentum space. Since this is a single object, momentum conservation requires a radial symmetry with respect to the normal space of the base vector set. This is even more stringent than the requirement of radial symmetry of simple two- component systems such as the Hydrogen atom. That the new term may lead to a self- confined purely electromagnetic particle is the second result of this paper.

The conditions on the form of this vortex lead this object to be charged. The picture is that in the pair creation process the scalar γ_P, initially zero everywhere (or constant) for the photon description, forms two non-zero pools of opposite sign pivot. This pivot arises in the present formalism from the product of the fields. Note that, for the purposes of this paper, it is only necessary that pivot arises in this process. The physical creation could also arise, for example, from the cancellation of field components (i.e. their subtraction), with pivot arising as necessary for mass-energy conservation. Once created, pivot acts to turn the propagation direction of the remaining fields to form two patterns circulating in one of the derived spaces (that of energy-momentum density). In this space, the twist of the fields about the propagation direction combines with this turn to result in a field distribution which, observed from outside, has the electric field either outward or inward directed. That is, this circulation leads necessarily to a pair of regions of opposite electric field divergence and with equal and opposite half-integral spin. These constitute the two equal and opposite charges identified as a particle-antiparticle pair[4].

Note carefully that none of the dimensions in the diagram are those of space or of time. They are all in the even set of derived dimensions, the fields, the momenta and the pivot. The solutions, likewise, are not in space and time. The extensions in the diagram are in bivector and scalar space. The major axis of the torus is (inversely) related to the strength of the scalar pivot term. This means that, though the element γ_P is a point as opposed to a line or a plane, it has a length scale associated with it related to the Compton wavelength.

4 Conclusions

A model of fermionic, half integral spin field configurations with a double-covering topology has been proposed for the electron and positron. A consideration of the overlap of such particles allows a physical understanding of the origin of the Pauli exclusion principle. A new kind of scalar quantity, called the pivot (because of its effect of turning the direction of propagation of electromagnetic momentum) has been introduced. This is proposed to account for the self confinement of electromagnetism into toroidal vortices in momentum space. The simplest such objects are identified with the electron and positron.

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