Quantum Spin Coherence In Four Derived 3-Spaces

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ABSTRACT:

A new theory is proposed that integrates sub-quantum mechanics — a development of relativistic quantum mechanics and electromagnetism — in a way that allows new insights into the modeling of electrons in atomic and molecular orbitals. Central to this approach is the mathematical treatment of each aspect of a system's energy flow in its appropriate space-time components, as well as the fact that the internal wave nature of energy and matter results in quantum systems with a coherent, resonant, harmonic nature. It is this wave nature that lies at the root of quantisation itself. This paper presents an approach to the modeling of a quantum system in a more complete way. It is proposed that derivatives in space-time lead to four 3-dimensional 'dynamical spaces.' This proves significant because these derivative spaces describe physical observables such as spin, electric field, and magnetic field. These have different properties, and this approach allows each to be considered either separately or together. The four 3-component derivative spaces are: energy (or frequency) space, electric field space, magnetic field space, and spin (or angular momentum) space. At the root of each of these are the properties of the *one* underlying 4-dimensional space-time. In particular, emerging from this new theory is a new way to model quantum spin and its contributions to energy, harmonic stability, and quantisation. This approach allows a clearer insight into the nature of electron pairing, as well as the reason that electrons can pair up at all, in spite of their charge repulsion.

Keywords:
quantum spin, harmonic resonance, field cancellation, relativistic quantum mechanics, cooper pair

1. WHY A NEW APPROACH?

Despite significant progress in the last century in quantum mechanics and the physics of materials, we still have unanswered questions about the nature of material particles, the behavior of materials, and the reasons behind certain chemical processes. One limit to our understanding is imposed by our lack of insight into what an electron is and how it interacts at the scale of the single electron. This dictates how it behaves in an electron shell, in electron bonding, and in crystal structures. A better understanding of the nature of an electron and its internal behavior at sub-electron length scales would enable a host of improved applications. These include better engineering in the materials science of high temperature superconductors and better explanations of the basis of the magnetic properties of materials. This is made possible by an advance in the theory of relativistic quantum mechanics[1-7], which, among other things, overcomes a constraint present in the Dirac equation by treating the rest-mass term as an integral part of the system dynamics.

We propose that key insight into the behavior of electrons at electron length scales can be obtained with a better understanding of their internal behavior. In the new paradigm, it is the internal dynamics of particles and their inter-actions that give rise to aspects of the quantisation of charge and spin. We refer to this as 'sub-quantum mechanics.' Sub-quantum mechanics develops and extends relativistic quantum mechanics into a realm that was previously taken to be simply axiomatic (e.g. "electrons have spin"), and therefore inaccessible. This more penetrating understanding, using the more advanced theory, allows a new way of thinking about the internal dynamics of elementary particles, atoms, and molecules, as well as how they interact with electric fields, magnetic fields, and quantum spin. Simply put, it enables thinking about materials and particles in ways that could not otherwise be thought about at all. In particular, sub-quantum mechanics provides key insights into the conceptual role of quantum spin, which will be explored in this paper (*see §2.1.4*.), as well as in future work[16].

1.1 The New Approach

The substructure of the electron, upon which this paper is based, has been described in more detail in Williamson and van der Mark[2,4]. It will be further expanded upon in the next paper in this series[16]. These papers describe the electron and positron formed by the condensation of two photons of the appropriate energy in a pair creation process as a pair of self-confined, toroidal, double-loop oscillations. The internal motion of each particle may be described as a "quantum bicycle," or "quicycle." What is it, exactly, that is 'cycling' in this *quicycle*? Williamson and van der Mark suggested in 1997[4] that the internal wave function of each elementary particle may be thought of as a circulating photon in perfect harmonic oscillation with itself as it completes two revolutions for every one wavelength.

A new feature[2] of this new theory is that, in addition to field, this harmonic coherence expresses explicitly through two rest-mass like terms. Though the particles themselves arise from pure field and still contain aspects of field, they are afterwards wholly an electron and a positron.

The initial photon energy can then be thought of as flowing dynamically not only through its electric field and magnetic field components, but also explicitly through those of spin, charge, and mass of the subatomic particle. In the theory, the rest mass terms arise through the cancellation of some of the initial field during the pair creation process. The new stable coherence that is the isolated electron is an intimate and dynamic flow between current, spin, and charge on the one hand and field and the pair of rest-mass terms on the other. The electron's internal motion can therefore be thought of as that of a self constrained electro-mass-magnetic wave with stability and a rest-mass core.

The key point of relevance for this paper is that the electron is a dynamical object with internal motion, a *quantum bicycle*, that is stable in its dynamics. An electron has charge precisely because it is a stable, local centre of inter-action [7]. An electric potential is inherent in the dynamics of an electron's internal structure, the *quicycle*. The equations for the interactions between charges were laid out by James Clerk Maxwell in 1865[17]. The new theory encompasses and expands on these.

1.2 Root-Energy & The Basis Of The New Approach

The main feature of this paper is a detailed description of how space-time may be thought of in such a way that treats each component of a system's 'root-energy' in a more interpretable mathematical form. While not necessarily known by this term, 'root energy' is a recognized concept, referring to quantities that need to be 'squared' in order to represent an energy. Examples include the electric field[1] and the quantum mechanical wave function, Ψ.[2] Upon this foundation of rigorous constraints, the cancellation and coherence interactions between quantum states, those that result in the stable, resonant, coherent, harmonic states we perceive in the matter around us, can be more fully understood.

In quantum chemistry, atomic and molecular orbital wave-functions are extrapolations of the wave-functions associated with the single-electron hydrogen atom. As such, for any system involving more than one electron, simple combinatorics of these wave functions can never be more than first-order approximations[1,15]. In addition, the theory used is non-relativistic, necessitating the introduction of a set of ad-hoc "relativistic corrections." The model proposed in the present work, in contrast, enables an absolute relativistic, correction-free treatment of multi-electron systems.

The full mathematics behind the *quicycle's* photonic double-loop rotation integrates relativistic quantum mechanics and electromagnetism in a way that overcomes key shortcomings of the state of the art. Dirac's theory of relativistic quantum mechanics is not up to this task for two primary reasons. Dirac introduced the rest mass as an inert parameter[3], and the electromagnetic field through the vector potential in the 'minimal coupling' scenario. The upshot

[1] $E_{field} = \frac{1}{2}\varepsilon\mathcal{E}^2$, where $\mathcal{E}$ is the electric field.

[2] In Quantum Mechanics, energy density is proportional to $\Psi^{\dagger}\Psi$.

is that the Dirac theory is not very useful for imagining the proper spaces of spin and field and their relationships to one another. This has introduced confusions which have been only partially resolved, and have led to the Dirac theory not having proven very useful, practically. This is why the non-relativistic Schroedinger theory has been used overwhelmingly in chemistry and engineering until now. Though the non-relativistic theory has proven very useful, it has several limitations, not the least of which is that the complex space in which it is formulated is too simple to properly describe the non-commutative nature of space-time. Neither Dirac nor Schroedinger thus prove completely adequate to properly describe the full interplay between mass, current, field, and spin.

This new paradigm features an electron that is an object whose underlying nature is electro-(rest)mass-magnetic, containing a self-confined photon[2,4,7]. This lies at the root of its electric charge, magnetic field, wave properties, and quantum spin. Each of these energy elements, including the mass-energy, flows dynamically into and through the other, being part of the same harmonic quantum system, and this is what gives rise to interactions such as spin-spin and spin-orbit couplings, for example. But spin has importance well beyond simply being a component of system energy. It is one of the primary elements behind the Exclusion Principle, and its non-negotiable conservation of angular momentum means that spin can accord a coherent stability even if it involves an *increase* in energy[12].

This new approach is aimed at, on the one hand, reducing the reliance upon approximation in wave-function modeling and to more completely model electrons, their interactions, and their consequent orbital structures, and on the other hand, to provide a new conceptual framework for the further development of new materials, devices, and systems. A full set of wave-functions describing these spaces is under development.

Conventionally, electric and magnetic field have been thought of as arising from the 4-differential of a 4-vector[3] potential. In the new theory, as well as in the six components of field, the four differential leads to 10 further components, including mass, spin, and charge, as will become clear once the mathematics is laid out more fully below (in *§2.1.5*.). The differentials yield four groups of three dimensions and four groups of one dimension, for a total of sixteen dimensions of reality. Bookkeeping behavior in these sixteen dimensions is not only useful to our physical interpretation of the model, but critical to accurately modeling the behavior of electrons as (and in) dynamical systems.

For short reference, the four groups within these sixteen dimensions are reprinted here. The four 3-dimensional elements are: space (*x, y, z*), electric potential (E_x, E_y, E_z), magnetic potential (B_{xy}, B_{yz}, B_{zx}), and spin (S_x, S_y, S_z). The four 1-dimensional elements are:[4] Ξ_P (‘pivot’), Ξ_o (time), Ξ_{123} (‘hedgehog’), Ξ_{0123} (‘quedgehog’)[2,18].

This paper tackles each of these sets of dimensions (or ‘spaces’) in turn, providing guidance and intuition as to how the energy of an electron flows dynamically through each. It

[3] A 4-vector is a directed line element in the 4 dimensions of space-time.

[4] The Greek letter Ξ is pronounced “ksi.”

will also clarify how these dynamical elements are reflected in the expansion of the Williamson equation.

2. THE SPACES

The new theory approaches the universe in such a way that treats each component of a system's root-energy in its correct mathematical form. As such, this paper may speak in terms that seem unfamiliar in the context of traditional approaches, and some of these elements should therefore be clarified.

2.1: Four 3-Component Derivative Spaces

In traditional physics and chemistry, approaches tend to be focused in 3-dimensional space, though it is accepted that spacetime is 4-dimensional. It contains three dimensions of space and one dimension of time. But these four "components" each come into play in different combinations of x, y, z, and t, depending on which aspect of energy or field is being considered. We also encounter the inverses of these four components, for example frequency (or energy) is measured in *Hertz* (or sec^{-1}), which is *inverse*-time (t^{-1}). By way of another example, the physics of spin is dealt with differently in mathematics than the physics of electric field. Electric charge is part of a 4-vector, electric and magnetic fields are parts of a bi-vector[5], and spin is part of a tri-vector[6], each derived as products and quotients of the base spacetime components. In order to deal with these more precisely, we consider that there are four superimposed derived[7] 3-dimensional spaces, each relating to a different aspect of the energy content of the system, and together, encompassing it all into one overarching dynamic coherence[7]. While this construct of four superimposed 3-spaces is indeed useful in order to treat the mathematics of the physics accurately, in reality there is only *one* underlying 4-dimensional spacetime. The other spaces are derivatives of this, in both the literal and mathematical senses of the term.

When dealing with energy waves and their resonant and harmonic interactions, it is pertinent to ask *'what* is waving?' and 'what is it waving *in*?' The answer is that, since these systems are made up of energy in the forms of mass, electric field, magnetic field, and angular momentum, their energy is waving in and out of four superimposed 3-component derivative spaces. These spaces are space-space (or frequency space), electric field space, magnetic field space, and spin space, and they exist together in a harmony of phases[13,14]. Each will be expanded upon below.

In simple systems, in each of these 'spaces' energy will manifest spherically since each has only one center — a center of mass, charge, magnetic moment, or angular momentum[4].

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[5] A bi-vector is a directed plane element.

[6] A tri-vector is a directed volume element.

[7] As in both the conventional and the mathematical sense of 'derivative.' Taking a linear derivative changes the number of dimensions by one, for example projecting a 4-dimensional reality onto 3 specific dimensions, depending on which derivative is taken. Similarly, integrating can also change the dimension.

This gives rise to spherical harmonic solutions. Its mathematics dates back to the polynomial work of Legendre and Laguerre, but as we will see, in order to encompass the physics of these spaces correctly, the 'normal' complex mathematics of quantum mechanics is not sufficient.

A basic quantum mechanical wave can be represented in a complex space by a wave function such as $Ae^{i(kx-\omega t)}$. The very form of this equation incorporates both space and time, and implies that the wave is waving in space (x) and time (t). In this equation, k represents a spatial frequency, an inverse-length. Inverse-length (k) multiplies with length (x) to produce unity — no net change. (Inverse-length multiplied by length results in a scalar term.) The ω represents a temporal frequency, an inverse-time. Inverse-time (ω) multiplies with time (t) to produce a scalar wave phase. The wave-function is a statement in mathematical terms of a dynamical wave waving between space, inverse-space, time, and inverse-time.

However, a complex mathematics with only two components, one real and one imaginary, does not suffice here: complex it may be, but it is not properly complex enough. In the new theory, waves must have at least four components in order to encompass the complexity of 4-dimensional reality, each element able to flow into and through the other, even as each transforms slightly differently in the (non-commutative) mathematics (as we will see below).[8]

In the 'hypercomplex' Clifford algebra employed in sub-quantum mechanics, there are 16 components that encompass these four superimposed 3-spaces and four connected single-element 'spaces'[2]. While these 16 components are linearly independent, they are related through a set of linear differential equations. There are many similarities to Dirac's quantum mechanics, though, notably, mass is present in the new theory as a dynamic (scalar) energy component, as mentioned above. Although this mathematics has 16 degrees of freedom, it describes a single overall dynamic process whose net energy (or, more properly, square-root energy) change is zero (as detailed in the above mentioned Williamson equation, $\mathscr{D}_{\mu}\mathbf{\Xi}_{\mathscr{G}}=0$). The equations of electromagnetism are special solutions of the Williamson equation (as they would have to be). Thus, while electric field energy may fold into spin energy, which may flow into mass-energy, for instance, the overall result for the system is (and must be) a stable, coherent, dynamic, and harmonic energy state. Quantum mechanical waves in the new paradigm are thus not so much *in* space and time, as they are *of* (inverse) space and time.

This may sound very complex, and it is indeed much more complex than the 'complex' algebra used in conventional quantum mechanics, but herein lies its beauty and utility. In reality the spaces derived from 4-spacetime are divided into a set of four 3-component 'spaces'. Three-dimensional spaces are easier for the human mind to imagine, develop, and engineer with than are 4-dimensional spacetimes. In the following, this utility will be exploited to better understand the geometrical relationship in matter as it manifests, with particular reference to the electron.

[8] The mathematics that represents physics must adequately account for the handedness innate to many natural systems. Ordinary complex numbers do not achieve this, and it necessitates an anti-Euclidian (– – –) metric.[19]

2.1.1. Space (or Frequency) Space

Space-space is in the vector realm. The underlying nature of this space is a directionality (x, y, and z), and each of the three components involves only *one* of the spatial dimensions.

While space and time are, of course, both of crucial importance, the more fundamental 'space' for quantisation is the symmetric inverse — frequency space. It is in this space, for example, that the relationship between energy and temporal frequency $E=\hbar\omega$, and momentum and spatial frequency $\boldsymbol{p}=\hbar\boldsymbol{k}$, are defined. These spaces are the basis from which the other spaces are derived, in both the usual and the mathematical sense. They contain the four dimensions of space and time, which appear as differentials (a special form of inversion) in the dynamical equations of sub-quantum mechanics. In this sense, the dynamics of all interactions are woven of space, time, and their (differential) inverses[7].

The mathematical derivatives express dynamical transformations, which in a real sense, unify the different aspects of the resonant harmonic energy flow through its various forms. It is in this way that the overarching constraint of the conservation of energy manifests. Harmonic resonances occur in frequency space. Frequencies are exact because they encompass the conservation of energy. A fundamental frequency is required in order to sustain and lock coherent resonance structures between space, time, field, and spin. Consequently, each of these dynamical states have exact energies associated with them. The electron, for example, resonates at an energy of 511 keV, and the electron and proton in the hydrogen atom resonate mutually at 13.6 eV, which represents their binding energy. These energies have characteristic frequencies related to each other by Planck's constant. At the quantum scale, frequency *is* energy, as expressed in Planck's equation $E=h\nu$.

2.1.2. Electric Field Space

Electric field space is the realm in which particles and atoms bind. This is the 3-space which largely defines the structure of atoms, molecules, and crystals, and is, primarily, the reason the material universe appears 3-dimensional to our senses. Our visual perception of the physical universe occurs largely in this space because we cannot 'see' time. Our images are thus a 3-dimensional projection — a collection of dynamic structures that we perceive as an average over time. While the electric field is the weakest of the four interactions, it occurs over the longest range. Up close, it may be dominated by other considerations, such as magnetism or spin, but at a distance it returns to primacy.

In chemistry, the electron geometries and orbital images we encounter occur in electric field space. The nodes (or boundaries) between orbitals are nodes in the electric field as a result of the addition of fields that gives rise to electric repulsion. Note carefully that this is in the 3-dimensionality of field space, not that of space-space. Electric field space is derived from space-space. The underlying nature of the (three component) electric field is that of the rate of change of space by time (dx/dt, dy/dt and dz/dt).[9] Electric field is therefore a bi-vector quantity.

[9] Combinations such as dx/dt and dt/dx project onto each other in the physics of electromagnetism.

The physical effect of an electric field on a charge is to accelerate it linearly. Although these three components involve all four variables *x*, *y*, *z*, and *t*, there are only three quotient components in the electric field, and each involves *one* of the spatial dimensions and time.

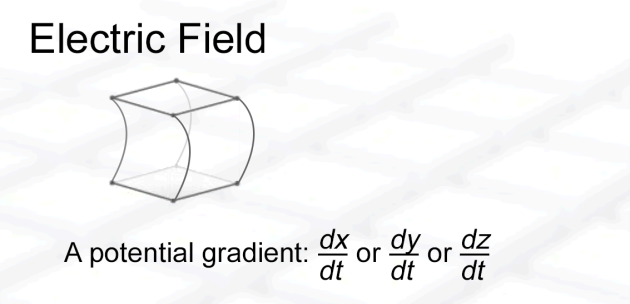

Figure 1: The electric field transformations

Electric field space is thus related to space-space, but it involves *space* multiplied by *inverse-time* (and *time* multiplied by *inverse-space*). Electric field space is, and always was, 3-dimensional, not 4-dimensional. Four dimensions cannot be portrayed in three just as the passage of time cannot be portrayed on a static drawing surface. Any images we contrive are therefore just convenient representations of an aspect of the harmonic system, one shade of a dynamic reality.

2.1.3. Magnetic Field Space

Magnetic field space is part of the realm of internal coherence within a particle. Whereas the underlying nature of the (three component) electric field is that of the rate of change of space by time (*dx/dt*, *dy/dt* and *dz/dt*), the magnetic field is that of the rate of change of space by perpendicular space (*dx/dy*, *dy/dz* and *dz/dx*). Magnetic field is therefore also a bi-vector quantity. A partial analogy is that of a spatial torsion or 'twist-bias,' though not a rotation since there is no time (*t*) component in these derivatives. While the physical effect of an electric field on a charge is to accelerate it linearly, that of the magnetic field is to make it go round and round in circles. This is a fundamentally different type of transformation to that experienced in electric field space. These three components involve only the three spatial variables, *x*, *y*, and *z*. There are only three quotient components in the magnetic field, and each involves *two* of the spatial dimensions.

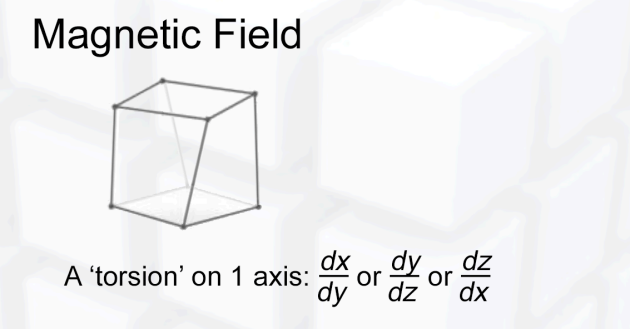

Figure 2: The magnetic field transformations

Indeed, in field space, the Maxwell equations and the new equations are, term for term, identical (*see §2.1.5 below*), so this is to be expected. This is as it must be: new theories must parallel that which has gone before in the appropriate limits.

2.1.4. Spin Space & Quantisation

The new theory of sub-quantum mechanics can be extended into the arena of atomic and molecular structures, with particular focus on the coherence and cancellation interactions occurring in the electric fields, magnetic fields, and spins of adjacent electrons. This approach is here dubbed 'sub-quantum chemistry,' and it enables a new way of thinking into the quantum mechanics of chemical systems. One key result of this extended way of thinking (and of this paper) is that it allows the very nature and cause of quantisation to be understood more clearly. Quantisation is not merely a set of axioms. It is a consequence of underlying physical processes, emerging from the fact that energy and matter possess an intrinsic wave nature. Waves necessarily contain boundary conditions. Quantum spin, like all stable harmonic wave resonances, must be quantised. Put simply, a rotation through 2π (or 4π for fermions) must bring one back to the starting point, which must constitute an integer number of wavelengths. *This is the proper origin of quantisation in quantum mechanics.*

For any coherent system, each element of field, spin, and mass-energy flow must attain harmonic phase coherence, both with itself and with every other element. The total energy of a system may then be viewed either as an integration over the total field energy density, or as a quantum energy based on the frequency through $E=h\nu$. One can consider the energy either in spin-flow or mass-field spaces, but not both[10].

Spin space involves a different type of transformation than magnetic or electric field space, and is more difficult to visualise simply. Spin may be understood as a dynamical rate of change of a rate of change. The angular momentum is the rate of change of momentum with respect to perpendicular space, and the mathematics will take the form of $d/dx(dy/dt)$. Spin is therefore a tri-vector quantity. Although these three components involve all four variables x, y, z, and t, there are only three quotient components in spin, and each involves *two* of the spatial dimensions and time.

This may be visualised as the rate at which the "y" momentum is changing in the "x" direction and represents a circulation in the xy plane. We refer to this as the "z" angular momentum, but the flow occurs in each of the other dimensions — x, y, and t — but specifically *not* in z. Spherical systems, whether an atom or the charge field of an electron, have no predetermined z-direction. In fact, spin is spherically symmetrical in 3-dimensions and has no z-axis until it is measured. It is justified, however, that these 3-spaces are related to z in that they have the same properties under rotation, and hence retain their apparent projection under such transformations. It is in this sense that they appear "superimposed"[8].

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[10] Since they are related by linear differential equations, the total energy may be integrated in either field-mass space, or spin-current space, but because these are in 'perpendicular' spaces — one even, the other odd — adding both would be double-counting.

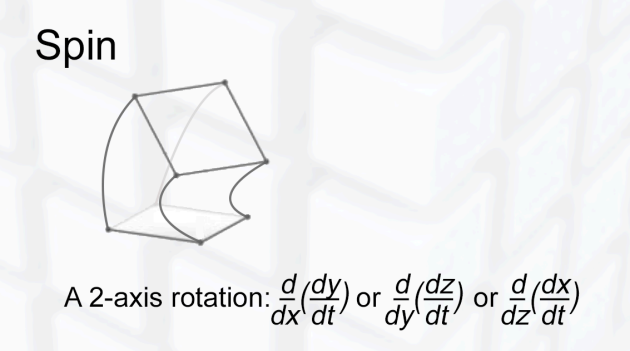

Figure 3: The spin transformations

In the new theory, spin is therefore more complex than merely an axial 'vector' (such as the magnetic field). It embodies a kind of field of circulation, where each point in that distribution is 'spinning' in the same sense, as if its local spacetime has the same 'spin-bias.'

It should be noted that the frequencies in real systems correspond to rotational frequencies rather than to back-and-forth vibrational frequencies because there is no boundary other than that of the particle itself[18]. One of the benefits of such a consideration of spin is that it opens the door to further discussion about the structure or specific topography of spin within a complete, coherent, harmonic quantum system.

Sub-quantum chemistry also allows the consideration of the interactions between adjacent subatomic particle spins, and these too must yield a stable, phase-locked state. When electrons with parallel spins interact, for example, they will either yield coherence or anti-coherence in spin space, depending upon their relative orientations[11], as well as their separation distance. Spin coherences, being quantised, will only exist in certain discrete configurations, in which adjacent spins are able to lock into one another, achieving phase coherence as they merge into a single, stable, harmonic, quantum spin state, characterised by the energy (frequency) of the coupling. A slightly longer or shorter coupling distance will not be permitted since small perturbations from a quantised spin coherence will lead to a loss of coherence — a dissonance. In energy terms, the underlying wave interference will cause a large increase in energy, even for a small perturbation (*see diagram below*). An "allowed" quantised spin coherence is here dubbed a Goldi-Locks Coherence, and will also be referred to as Parallel Spin Bonding, PSB, in future work.

[11] Axial versus equatorial with respect to their magnetic moments.

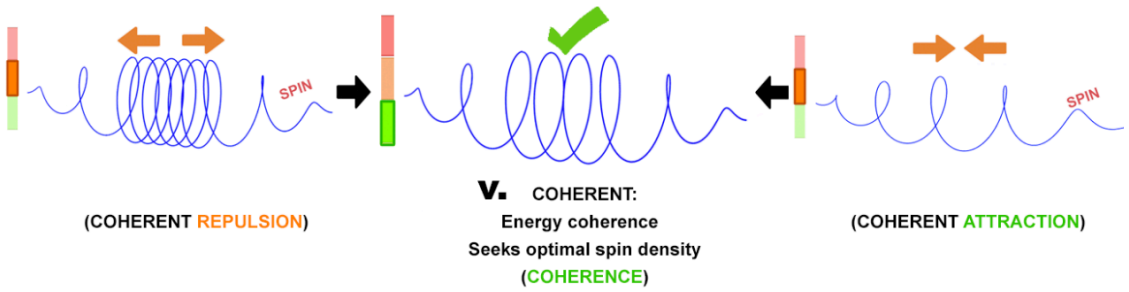

Figure 4: The Goldi-Locks Coherence in frequency space diagrams

The diagram above is designed to represent the energy of a quantum interaction. It uses the idea of a helical wave (or spring) *as an analogy only*, and should therefore not be interpreted too literally. It is simply an added visual tool, useful because energy can be represented differently in each of the derivative 3-spaces in ways that are difficult to illustrate in a two dimensional diagram.

It is the underlying physical effect of such spin coherences that is behind the "rules" often taken as a starting point for describing such quantum systems. It is proposed that this Goldi-Locks Coherence is responsible for the remarkable stability of degenerate electrons within an atom (or molecule), and it thus provides the 'why' behind, among other things, Hund's Second Rule. There may be, and in general are, other states close to or degenerate in energy to any particular quantum coherence, but they will have different discrete values in spin space. It will require a "quantum jump" to transition from, for example, the triplet to the singlet state of such a system. The local coherence and continuity of any given state comes not merely from energy considerations, but also from those of the conservation of momentum, field symmetry, and angular momentum. There are phase relationships in spin, momentum, and frequency which must combine to phase match in a "harmony of phases"[13,14].

Unlike the normal spatial rotations of a rigid body, the flow of quantum spin is 'rotating' in the same direction in all parts of the 'spin field.' The spacetime on all sides of the object possessing the spin will have the same 'spin-bias.'[12]

As will be further detailed in future work, another interesting difference between magnetic field and spin interactions is that, as opposed to the field cancellation that occurs in magnetic field space, in spin space, the superimposition of equal, antiparallel, and opposite-spin quantum angular momenta allows them, in theory, to *intertwine* with one another, here dubbed 'interweaving.' This would optimise spin density within the overlapping volume of (spin) space, lowering energy. It is, in fact, directly analogous to the way we consider the completely superimposed spins of antiparallel electrons resonating in the *di-electron* state (*as we will see in §4 below*). A partial overlap would result in a partial though still attractive interweaving coherence.

[12] An integration over this internal angular momentum field gives a net result at the outside boundary, but introduces a special spin (z-)axis, as discussed above, hence the reason L^2 and L_z are 'good' quantum numbers for the distribution as a whole in this special case.

Unlike in the magnetic field case, the nature of spin interactions are also expected to be more similar than different when comparing the axial and equatorial orientations because spin is an internal, quantum property.

If this conjecture is correct, spin is not canceled in this interweaving process; it is all present, though woven together in such a way that balances both spins when they occupy the same space. (Another way to think of it is that such overlapping spin space can support electrons of either or both intrinsic spins.) Interweaving thus creates the spin equivalent of counter-rotating toroidal vortices within the same volume of space. It must do so, in fact, since the spin of the underlying light-speed photons comprising the subatomic particles cannot be nullified, nor their motion stopped. They must rather attain resonance. The effect, however, is that overall spin *would* be reduced — since quantum spin is a measure of *overall* spin. Energy might thus be lowered, even as the two component spins remain present. The optimisation of spin volume may therefore amount to a more attractive energy state, despite the apparent anti-coherence of the antiparallel spins.

It should be noted that spin forces are not necessarily weaker than electric forces, but will have a far faster fall-off with distance. In chemistry, however, those manifesting between molecular electrons that are usually further apart than atomic electrons will express as weaker interactions.

2.1.5. The Mathematics Of Absolute Relativity

The Williamson equation (1) employs a Clifford-Dirac algebra in order to represent an absolutely relativistic set of coupled linear differential equations that allow each of the ‘spaces’ mentioned above to be identified discreetly in wave function modeling[2] of all aspects of subatomic particle and photon systems. The equation of motion for a non-interacting system is

$$\mathscr{D}_{\mu}\mathbf{\Xi}_{\mathscr{G}}=0. \qquad (1$$

The four-differential of the 16 component general multi-vector of equation (1) gives:

$$\begin{aligned}
&\mathscr{D}_{\mu}\mathbf{\Xi}_{\mathscr{G}}=\mathscr{F}_{\mathscr{G}}=\alpha_0(\partial_0\xi_P-\partial_1\xi_{01}-\partial_2\xi_{02}-\partial_3\xi_{03})+\\
&\alpha_{123}(\partial_0\xi_{0123}-\partial_1\xi_{23}-\partial_2\xi_{31}-\partial_3\xi_{12})+\alpha_1(-\partial_1\xi_P+\partial_1\xi_{01}-\partial_2\xi_{12}+\partial_3\xi_{31})+\\
&\alpha_2(-\partial_2\xi_P+\partial_0\xi_{02}+\partial_1\xi_{12}-\partial_3\xi_{23})+\alpha_3(-\partial_3\xi_P+\partial_0\xi_{03}-\partial_1\xi_{31}+\partial_2\xi_{23})+\\
&\alpha_{023}(\partial_0\xi_{23}-\partial_1\xi_{0123}+\partial_2\xi_{03}-\partial_3\xi_{02})+\alpha_{031}(\partial_0\xi_{31}-\partial_2\xi_{0123}-\partial_1\xi_{03}+\partial_3\xi_{01})+\\
&\alpha_{012}(\partial_0\xi_{12}-\partial_3\xi_{0123}+\partial_1\xi_{02}-\partial_2\xi_{01})+\alpha_P(\partial_0\xi_0+\partial_1\xi_1+\partial_2\xi_2+\partial_3\xi_3)+\\
&\alpha_{0123}(\partial_0\xi_{123}+\partial_1\xi_{023}+\partial_2\xi_{031}+\partial_3\xi_{012})+\alpha_{01}(\partial_0\xi_1+\partial_1\xi_0+\partial_2\xi_{012}-\partial_3\xi_{031})+\\
&\alpha_{02}(\partial_0\xi_2+\partial_2\xi_0-\partial_1\xi_{012}+\partial_3\xi_{023})+\alpha_{03}(\partial_0\xi_3+\partial_3\xi_0+\partial_1\xi_{031}-\partial_2\xi_{023})+\\
&\alpha_{23}(\partial_0\xi_{023}+\partial_1\xi_{123}-\partial_2\xi_3+\partial_3\xi_2)+\alpha_{31}(\partial_0\xi_{031}+\partial_2\xi_{123}+\partial_1\xi_3-\partial_3\xi_1)+\\
&\alpha_{12}(\partial_0\xi_{012}+\partial_3\xi_{123}-\partial_1\xi_2+\partial_2\xi_1)=0 \qquad (2
\end{aligned}$$

The translation of equation (2) to a more familiar form, with the proper 4-D multivector component written to the left, yields:

$$\begin{aligned}
&\alpha_0(\vec{\nabla}\cdot\vec{E}+\partial_0 P)=C_0\alpha_0\\
&\alpha_{123}(\vec{\nabla}\cdot\vec{B}+\partial_0 Q)=C_{123}\alpha_{123}\\
&\alpha_i\left(\vec{\nabla}\times\vec{B}-\partial_0\vec{E}-\vec{\nabla}P\right)=C_i\alpha_i\\
&\alpha_{0ij}(\vec{\nabla}\times\vec{E}+\partial_0\vec{B}+\vec{\nabla}Q)=C_{0ij}\alpha_{0ij}\\
&\alpha_P(\vec{\nabla}\cdot\vec{A}+\partial_0 A_0)=C_P\alpha_P\\
&\alpha_{0123}(\vec{\nabla}\cdot\vec{T}+\partial_0 T_0)=C_Q\alpha_{0123}\\
&\alpha_{i0}\left(\partial_0\vec{A}+\vec{\nabla}A_0+\vec{\nabla}\times\vec{T}\right)=C_{i0}\alpha_{i0}\\
&\alpha_{jk}\left(\partial_0\vec{T}+\vec{\nabla}T_0-\vec{\nabla}\times\vec{A}\right)=C_{jk}\alpha_{jk}
\end{aligned} \qquad (3$$

In this representation of the (square-root) energy flow of the system, the terms that refers to spin, for example, are the α_{023}, α_{031}, and α_{012} terms (collectively, the α_{0ij} term). The translation of the spin component, with the proper 4-dimensional multi-vector component written to the left, is therefore given by:

$$\alpha_{0ij}(\boldsymbol{\nabla}\times\mathbf{E}+\partial_0\mathbf{B}+\boldsymbol{\nabla}\mathrm{Q})=C_{0ij}\alpha_{0ij}=0 \qquad (4$$

where **E** is the electric field vector, **B** is the magnetic field vector, Q is the dual (pseudo) scalar term that refers to the integral of spin[16]. Here, C_{0ij} represents a set of sixteen real number constants, here all zero. $C_{0ij} \neq 0$ is appropriate in the case of an interaction with an external system. Note that, in the absence of the "Q" term, this is exactly one of the Maxwell equations.

Similarly, within the context of this new theory:

- the α_{i0} term (or α_{20} term above) represents the electric field component
- the α_{jk} term (or α_{23} term above) represents the magnetic field component

Here, note that in the absence of the "T" terms, these are just the conventional constitutive equations of the electric and magnetic fields from the vector potential. The full mathematical details of this work are beyond the scope of this paper and can be pursued further into the referred work.

2.1.6. Dynamical Spatial Relationships

As stated above (in section 1.2), this new paradigm features an electron that is an object whose underlying nature is electro-(rest)mass-magnetic, containing a self-confined photon, which is what determines its properties. Each of these energy elements, including the

mass-energy, flows dynamically into and through the other, being part of the same harmonic quantum system. These dynamics are reflected in the harmonic Williamson equations of motion ($\mathscr{D}_\mu \Xi_\mathscr{G}=0$) described above.

The dynamical transformations engendered by the equations of motions under the 4-vector differential are illustrated in figure 5 below.

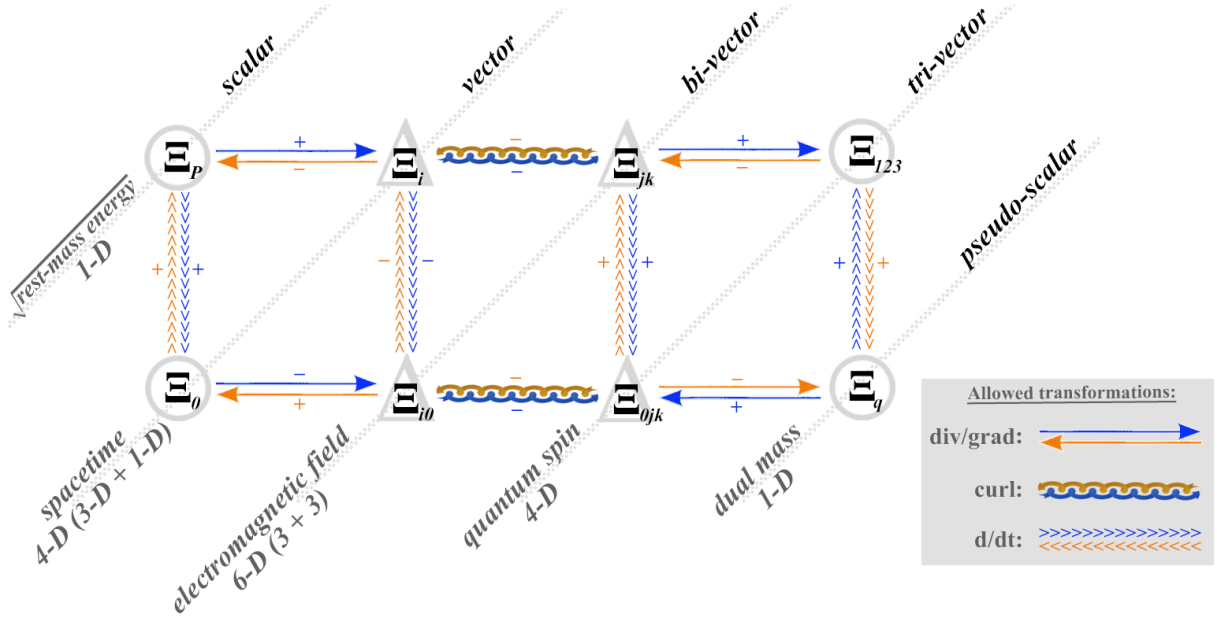

Figure 5: The dynamic transformations of energy flow through an electron. Triangles mean the term exists in three dimensions and circles meaning the term exists in a single dimension.

This diagram illustrates the dimensionality of each term (above) — scalar, vector, bi-vector, tri-vector, or pseudo-scalar — and the energy component that it represents (below) — rest-mass, (frequency) space, electric and magnetic field, quantum spin, or pseudoscalar (dual) mass. It also indicates the mathematical transformations that can take place in the energy flow from one element to the next.

Some loose physical interpretations of the different nodes of Figure 5 are as follows (for exact definitions refer to [2]):

- Ξ_P [Scalar] "Pivot" - Root mass-energy - the most basic building block
- Ξ_0 [Vector] Time or frequency (or energy when describing quantum systems)
- Ξ_i [Vector] Linear delineation of the fabric of space-time's space in *x*, *y*, and *z*
- Ξ_{io} [Bi-vector] Space-time acceleration - Electric Field
- Ξ_{jk} [Bi-vector] Space-space twist - Magnetic Field
- Ξ_{ojk} [Tri-vector] All three directions of spin
- Ξ_{123} [Tri-vector] Spherical directed radially in or out
- Ξ_q [Pseudoscalar] "Quedgehog" - 4-Spherical, directed radially in or out. Contributes to total rest mass.

Another way to look at how fields transform using the harmonic Williamson equations of motion ($\mathscr{D}_\mu \Xi_\mathscr{G}=0$) is to examine what happens when transformations are applied to known entities (each line between nodes in Figure 5):

Take, for example, an electromagnetic field. Figure 5 shows how this can be transformed into both quantum spin and linear current:

A magnetic field can be transformed into a portion of quantum spin through the change with respect to time. An electric field can be transformed into a portion of quantum spin through a change with respect to space. What that means physically, is that a varying electromagnetic field can create measurable quantum spin (since the same system can be viewed as one or the other, depending on the transform applied). These equations limit these transformations to a set of allowed values such as the discrete values of quantum spin (multiples of $\hbar$).

One can also apply a gradient transformation to an electromagnetic field to interrogate its contribution to the magnitude of linear current. This approach is commonly used in Maxwell's equations to find out the current in an electromagnetic system and is also a part of the Williamson equations. A 4-vector derivative of root-mass energy is another way to get an element of the 4 vector potential (Ξ_0, Ξ_i) .

A single dimensional example of a transform is applying the D_x/D_y transform (one dimension of curl) to the electric field in the x direction (E_x). This derives a trivector, a portion of the measure of quantum spin. A more full set of components of quantum spin would result from the curl on $E_{xyz}B_{xyz}$, d/dt of B_{xyz} and the gradient of B_{xyz}. The full quantum spin would be given by the entire set of Williamson equations to get the transforms of all three connections into the Ξ_{ojk} node.

To obtain the full value of the quantum spin and the linear current inherent in an electromagnetic system, the 4-derivative of the eight even components of Ξ (6 field components plus pivot and quedgehog) results in 32 components, which simplify into a 4-vector (current) and a 4 tri-vector (spin)[2]. The Williamson equations handle the application of a 4-differential onto the six dimensions of Electric and magnetic fields as a transform from one diagonal line in figure 5 to the next. Each of these transforms changes the nature of the description from even to odd, or odd to even (bi-vector to tri-vector, vector to scalar, etc.).

Maxwell's equations provide a mechanism for bookkeeping the transition of electric field to magnetic field and vice versa. This approach elucidates the nature of the stable dynamics of a photon (i.e. a traveling electromagnetic wave).

The Williamson equation extends electromagnetism, providing a mechanism for bookkeeping the transitions between the parameters defined in [2] and figure 5. This clarifies the nature of the stable dynamics of an electron, explaining how quantum mechanics works. In the same way that Maxwell's equations enable the entire modern field of electrical engineering, the Williamson equation will enable a new modern era of relativistic quantum engineering.

Dirac's equation is typically written as invariant matrices. *D/* operator: *d/dt(gamma 0) - d/dx(gamma 1) - d/dy(gamma 2) - d/dz(gamma 3)*. This is then a unit in covariant (gammas), contravariant (differential) form. The product of these two is then an invariant, a scalar. Then when you do a scalar derivative (as is commonly done), you get out the same sort of thing you put in. The common approach prevents the cross talk between nodes in Figure 5. (Complex vectors are used but still over constrain the mathematical description of the system).

2.1.7. What Is Waving In What?

Returning to the question of *'what is waving in what?'* that was posed above, the fields that are waving are not "in" space and time, they are "of" *space-and-inverse-time* and

time-and-inverse-space. They are waving in space and inverse-time, and time and inverse-space, and in the case of spin, in further combinatorics of these.

The present work is built upon a foundation that is a development of both classical electromagnetism and relativistic quantum mechanics. This paper is about understanding how things like field, spin, mass, and energy fit together in constructing the physical world in which we live. A complete explanation of them is beyond the scope of this paper. For the interested reader, far more detail can be found in the referred work [5,7,9,10]. That which is presented here is primarily intended to convey the key differences between the way each component interacts. It is about how they meld and mesh to produce the geometries of the material world that is of principal importance in this discussion.

2.2. The Dichotomy Of Spin

One of the enduring mysteries of quantum spin is its one dimensional nature. Spin is said to be either "up" or "down" for fermions, or to take on integer quantised values for atomic states. It is possible to understand this partially in terms of sub-quantum mechanics. If one has an element of spin in the "*z*" direction, it is, properly, in the directed volume element *xyt*. The proper integral of this about the *z* direction will yield a result with a four-dimensional form *xytz*. This has only two directions: namely an inward- or outward-directed 4-volume. An integer value of this corresponds to a full loop (or loops), as in the photon, and a half integral one to a nested loop of a loop, as in the electron (*depicted below*). (We will delve more deeply into the physical difference between the spin-up and spin-down electron and positron in future work[16].)

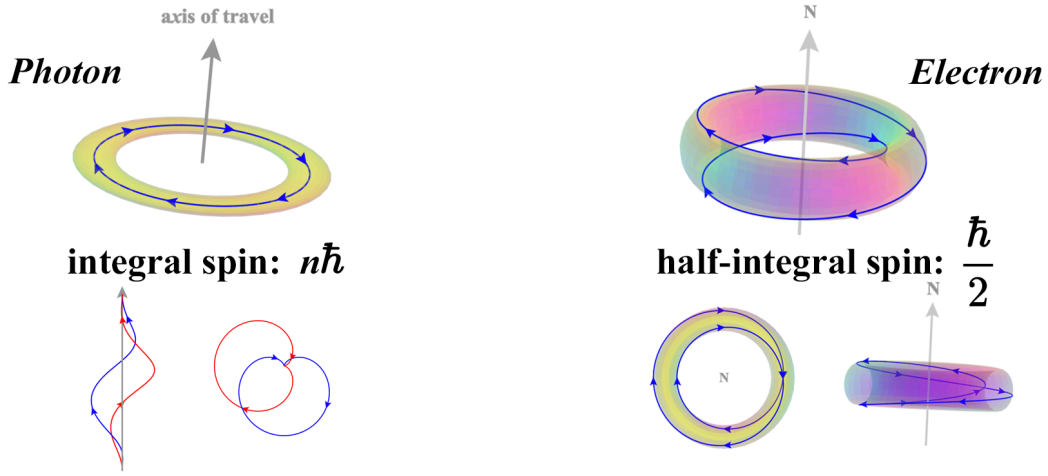

Figure 6: A graphical representation of integral spin (left), a single-loop per wavelength (such as a photon), versus half integral spin (right), a double-loop per wavelength (such as an electron).[4]

3. COHERENCE & CANCELLATION INTERACTIONS

A key conceptual focus of this theory centers around the coherences (or anti-coherences) and cancellation (or reinforcing) interactions that occur between harmonic elements such as field

or spin. Harmonic resonances are a consequence of the constructive (reinforcing) and destructive (canceling) interference that occurs when waves interact or overlap. These same interactions occur whether those waves represent electric energy, magnetic energy, angular momentum energy, or any other form of energy in a coherent harmonic resonance.

Harmonic coherences add stability and lower energy, even if a cancellation (of field) in the traditional sense may not be occurring, and they also add stability through a decrease in (spin) energy. By way of an example, when it comes to adding parallel quantum spin states, which are essentially angular momenta rotating on parallel spin axes in their proper space, their interaction will find their entire spin flow co-rotating, a favorable state, which leads to an enhancement of an atom's magnetic properties. Examples include ferro- and paramagnetism.

Cancellation interactions is a concept that has long been found in the accepted wisdom. We are familiar with it, by way of example, from the phenomena of magnetic attraction (in the case of magnetic field cancellation) and effective nuclear charge (in the case of electric field cancellation).

Magnetic attraction occurs when, for example, the axial magnetic lines of force emanating from two sources are aligned in the same direction. They are therefore able to resonate and cohere with one another, which allows for the lowering of energy through mutual field sharing. If they had been oriented in opposite directions, coherence would not have occurred. In fact, a coherent repulsion (an anti-coherence) would have resulted. (*See fig. 3.1 below.*)

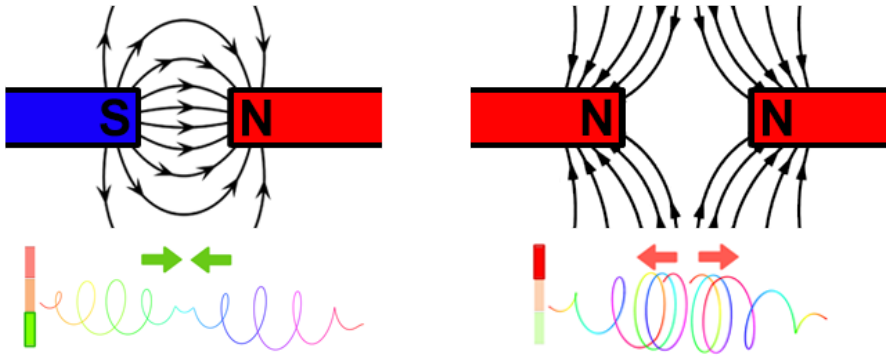

Figure 3.1: Magnetic field cancellation (left) and anti-coherence (right)

Another way to consider this field-canceling coherence would be to imagine that the 'north-facing field' of one is meeting the 'south-facing field' of the other. Since these oppositely oriented north and south fields annul one another when they cancel, energy is lowered. In either case, since lower energy is a more favorable state, an attractive force results.

Effective nuclear charge results from the imposition of charge symmetry, despite the small size of the nucleus compared to the electron cloud. The reason the valence electrons in the oxygen atom experience the attraction of a 6+ nuclear center is because the electric field of the 2 inner-shell electrons (*the di-electron*) precisely and completely field-cancels with the electric field of two of the protons in the nucleus. (We here ignore nuclear (hyper)fine structure for the purposes of this discussion.) The valence electrons therefore experience the atomic core as a

spherical center, made up of a nucleus and a *di-electron,* and carrying a charge of 6+. The concept of cancellation interactions explains the 'why' and the 'how' of the phenomenon of effective nuclear charge.

These effects are similar to some of those found in the Robinson Model of Nuclear Binding, which details charge and field interactions between nucleons.[11]

An important example of field cancellation occurs in the formation of the electron itself. As discussed above, the Williamson model of sub-quantum mechanics describes the internal toroidal topology of the electron. It finds the negative electric field directed radially outwardly at all points in order to give a negative charge profile for the particle. The magnetic field lies orthogonal to the charge, and given the electron's internal quantum spin, it is able to cancel against itself on average. This lowers the total energy dramatically, which is why the electron is such a highly stable particle. Since magnetic field is also two orders of magnitude stronger than electric field, if the electric and magnetic fields exchanged roles in order to produce a magnetic monopole with no net electrical charge, the mass would be far larger than the energy content of the system in the electric monopole state, which is why this high energy, magnetic monopole, state is unattainable for all practical purposes. This is the reason that bare magnetic monopoles are not found to exist in nature. If they could form, they would immediately decay into a much lower energy electric monopole.

Another important example occurs in the formation of the *di-electron*, the electron pair (or Cooper pair), which we will describe below (in §4). Additional examples will be discussed in more detail in future work on the topic of the hierarchy of electron interactions. These include *partial di-electron inclusion* (PDI), *linear spin bonding* (LSB), *parallel spin bonding* (PSB), and the above-mentioned *di-electron* formation, termed *total di-electron inclusion* (TDI).

Cancellation interactions are thus very important components of this story. They are necessary in order to understand why the inner structure of an electron yields a particle that has an electric monopole. More importantly for the present theory, they explain why and how two electrons can experience like-charge repulsion, yet be able to form strongly-bound and highly stable electron pairs within a nuclear charge well.

4. THE DI-ELECTRON

Arguably one of the most important cases of both field cancellation and spin coherence occurs during electron pair (*di-electron*) formation. When two electrons of opposite spin pair up, or when they envelop a helium nucleus, they will snap into an antiparallel relationship because this allows them to superimpose completely upon one another. This causes their magnetic fields to be oppositely aligned at every point, resulting in maximum magnetic field cancellation. Superimposition of the electrons also allows their antiparallel toroidal spins to become completely interwoven, optimizing their quantum spin (*as described above in §2.1.4*).

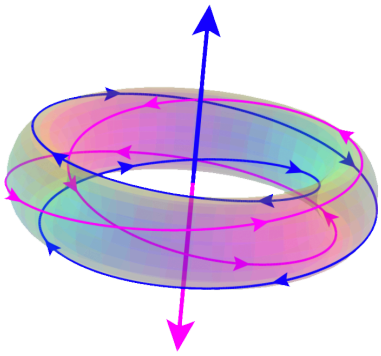

Figure 8: Spin interweaving in the di-electron

These interactions lower both energy and angular momentum *significantly*, forming a new mixed state — a *di-electron* boson. At such close distances, the attraction of magnetic field cancellation and spin coherence overwhelm the electrostatic charge repulsion[5]. The *di-electron* is, effectively, a new symmetric charge-2 bosonic state. It has an inner coherence which is not just that of energy but also that of spin. This may help to explain, for example, the persistence of *di-electrons* (Cooper pairs) in high temperature superconductors and the anomalous mass energy observed in some superconducting systems[12].

For more detail on the internal *di-electron* spin coherence, as well as the disallowed state of Pauli Exclusion, see future work[16].

5. CONCLUSIONS:

A new approach to the mathematical modeling of quantum systems has been proposed. One of the primary goals of this paper is to clarify not only the physics of quantum spin, but also how to view it and apprehend it in a mathematical context of electricity and magnetism that is inherently interpretable in its correct mathematical transformations.

Recent work in the field of sub-quantum mechanics has allowed us to think about processes in physics and chemistry in a more concrete way. It allows us to understand each component of matter, energy, and their interactions in its specific and correct combination of spatial and temporal dimensions, along with their appropriate mathematical transformations. The dynamics of space-time constitute four superimposed 3-component derivative spaces, namely energy (frequency) space, electric field space, magnetic field space, and spin (or angular momentum) space, and their associated single element spaces, namely mass-energy space, time, and the directed spatial and temporo-spatial volume elements.

As a consequence of the wave nature of matter and energy, cancellation and coherence interactions occur between resonant energy elements in and between each of these spaces. It is suggested that, while spin and magnetic field, for example, may have certain similarities and are related as differentials of one another, there are meaningful differences between the ways in which they interact. An example of such a difference is that charge and magnetic field

experience repulsion when their alignment yields anti-coherence, while certain spin states can yield either repulsion or attraction into specific quantised coherences.

If the proposed theory is correct, it would be possible for unpaired electrons to be stabilised (or partially-stabilised) through quantum interactions with other unpaired electrons, *without* having to pair up in the traditional sense. This approach also allows a clearer insight into the nature of electron pairing, as well as the reason that electrons can pair up at all, in spite of their charge repulsion.

In addition, quantum spin, though a part of the continuous energy flow through a system, can be evaluated as a separate harmonic coherence. This allows for the possibility of considering spin as an influencer, for example by explaining the spin-mediated, physical reason behind many axiomatic principles in chemistry, such as the Pauli Exclusion Principle and Hund's Second Rule interactions like Pauli Exclusion or Hund's 2nd Rule[16]. Quantum electron interactions and spin can also be evaluated in terms of their contribution to the magnetic properties of an element, such as paramagnetism and ferromagnetism. These concepts will be explored further in future work.

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