Calculating The Electron Charge From Theory Alone

The Idealized Williamson-van Der Mark Calculation

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1. INTRODUCTION

From the time of its discovery in 1887, no one could explain why the electron has the charge it has. We could only ever *measure* it.

Those who accepted the idea of it being a point particle were caught in a conceptual cul-de-sac: Because an actual point has no dimension, and therefore no structure from which to derive its properties, they had no way to approach a search for the source of charge.

That changed in 1997, when John Williamson (*CERN/Philips Research Labs*) and Martin van der Mark (*Philips Research Labs*) published their important paper [1], suggesting a toroidal momentum substructure for the electron, one which explained the actual physics behind both charge and spin beautifully and intuitively. *And* rigorously.

They conjected that an electron must be made of a gamma ray photon because that is what it turns into when it annihilates. And we know the exact energy of that gamma ray. All we need is to know the geometry of the photon's momentum when it is in electron form, and we can calculate the charge from theory alone. And that is what they did. Their value was in precisely the correct neighborhood — 10^{-19} Coulombs, although the number itself was off by 9%. They calculate 1.458×10^{-19} instead of 1.602×10^{-19}.

More notice should have been taken then, but 9% discrepancy may have given people enough room to ignore it. Fortunately, the model is finally beginning to gain the recognition it deserves now. We have also continued the work of Williamson and Van der Mark, here at Quicycle, and we will soon be releasing a refinement of their geometric model and charge calculation.

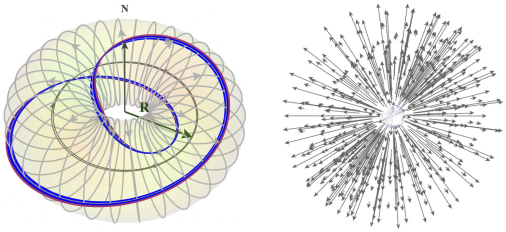

Figure 1: Schematic of the internal momentum-space flow in the WvdM model (left). The toroidal structure has a major radius of length $R = \lambda_C/4\pi$. The consequent charge field (right) is static and spherical [1].[1]

[1] Images generated using https://c3d.libretexts.org/CalcPlot3D/index.html

Below is a brief summary of their original calculation of the elementary charge, using nothing but the electron's Compton wavelength and their simplified momentum-space geometry.

2. THE CALCULATION

The WvdM charge calculation invokes only established equations from conventional physics. It can be summarized as follows:

2.1. Photon Energy In A One-Wavelength Knot

In the original WvdM calculation, for an *idealized electron*, it is assumed that the double loop rotation is made up of a single photon, with a wavelength equal to the electron's Compton wavelength, λ_C:

$$\lambda_C = \frac{h}{m_e c} \approx 2.42631023538 \times 10^{-12} m, \tag{1}$$

where h is Planck's constant, m_e is the mass of the electron, and c is the speed of light. The photon's energy, U, (in its closed-loop rest frame) is the same as if it were free:

$$U = m_e c^2 = \frac{hc}{\lambda_C} = 511\ keV. \tag{1a}$$

(Note: This model uses λ_C as an observational input at the Compton scale. However, the Compton wavelength ($\lambda_C = h/m_e c$) encodes the electron's rest mass, which was historically determined through electromagnetic measurements. This may raise the concern of using charge as an input parameter in the model's charge calculation.

We allay this concern in two ways. Primarily, we acknowledge that m_e, e, and a_e are all experimentally entangled through their electromagnetic origin. In this paper, our goal is to demonstrate that, within the construct of the WvdM model, the elementary charge is obtained using only λ_C, toroidal geometry, and a quantization condition. Secondarily, we note that the electron's Compton wavelength can also be known spectroscopically, via photon energy ($E = h\nu$ and $\lambda_C = c/\nu_C$) in pair production/annihilation measurements. In principle, λ_C *can* therefore be determined from pair-production thresholds, without reference to low-energy Coulomb measurements, so the charge itself is not being fed back into the calculation.

2.2. Volume of Confinement and Energy Density

For the purposes of their calculation, Williamson and Van der Mark chose a double-loop structure made up of two simple rotations — two roughly equal circular paths of about the same

length, $\lambda_C/2$. (This also describes a simplified and *very thin* torus.) Since this 'double circumference' must have a path length of $\lambda_C = 2 \times 2\pi R = 4\pi R$, the average major toroidal radius, R (which defines the average radius of energy transport around the electron) would then be

$$R = \frac{\lambda_C}{4\pi}. \tag{2}$$

The photon's electromagnetic field is thus spread within a highly localized region. In their charge derivation, Williamson and Van der Mark estimate that the electron's field must be confined within a spherical volume that has a rotational horizon radius of

$$r_{hor} = \frac{\lambda_C}{2}. \tag{2a}$$

This is because the "limitation of the speed of light means that only paths within this radius can provide a contribution to the circulation of energy inside the object" ([1], §3). This means, in the extremal case of a double loop, in which one loop has a tiny radius, very close to the axis of rotation, and in which the other loop is extremely elliptical — almost to the point of being a straight line — its length cannot extend farther than a distance of half the Compton wavelength from the center (as depicted in Figure 3).

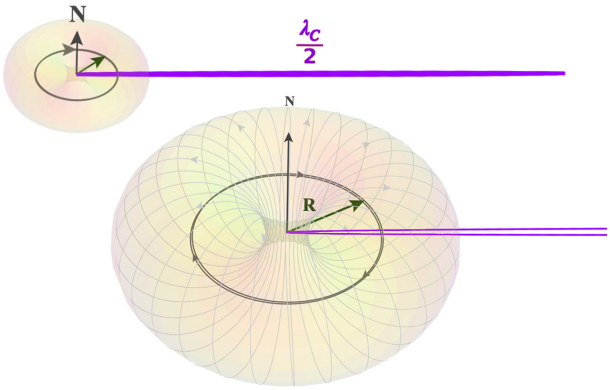

Figure 3: The (idealized) rotational horizon radius of the electron's electric field (purple) due to the range of rotational positions of the toroidal momentum topology, with a path length, λ_C, around its axis of rotation (N).

As such, the volume of the electron is based on a radius (a rotational horizon) equal to half of its Compton wavelength (λ_C). We can then use this radius to find the volume of the rotational part[2] of the electron's electric field, as follows:

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[2] The non-rotational energy beyond that point (to first order) is equal to the fraction $\frac{\alpha}{2\pi} \cdot U_{e-} \approx \frac{1}{861} \cdot U_{e-}$, where α represents the fine structure constant. According to the WvdM model, this energy partitioning is responsible for the leading term of the gyromagnetic ratio, *g-2*.

$$V = \frac{4\pi r_{hor}^3}{3} = \frac{4\pi(\frac{\lambda_C}{2})^3}{3} = \frac{\pi\lambda_C^3}{6}. \quad (3)$$

Note, again, that the electron's toroidal topology is in momentum space. In electric field space — the 3-dimensional (or 3-component) bi-vector space of the electric field — the electron manifests as a static, spherical charge field [1,2]. (This is the reason that we do not use a toroidal volume when calculating the electric field density, E. We are not calculating a 'momentum volume.')

The energy density of the electromagnetic field is given by:

$$W = W_E + W_B = \frac{1}{2}(\varepsilon_0 \mid E \mid^2 + \frac{1}{\mu_0} \mid B \mid^2), \quad (4)$$

where E and B are the electric and magnetic fields, ε_0 is the vacuum permittivity, and μ_0 the vacuum permeability. For an electromagnetic wave, the electric and magnetic field energies are equal (since in free space $E = cB$ and $c^{-2} = \varepsilon_0\mu_0$). Thus, half of the photon's energy is in the electric field and half in the magnetic field. This is intuitive since the internal photon's two field aspects, its electric field and magnetic field, exist in balance. In general, the topology rectifies one into charge and the other into confinement and magnetic moment, dividing the energy as:

$$U_E = U_B = \frac{U_{m_ec^2}}{2} = \frac{hc}{2\lambda_C} = \frac{511}{2}\ keV. \quad (4a)$$

2.3. Average Electric Field Within The Volume

Using the above, the average electric field energy density, W_E, inside the volume can be found, and from that, the average field magnitude can be deduced. The electric field energy density is given by:

$$W_E = \frac{U_E}{V} = \frac{U_{m_ec^2}}{2V} = \frac{1}{2}\varepsilon_0\langle E^2\rangle. \quad (4b)$$

where $\langle E^2\rangle$ is the mean squared field in the volume.

Equating these yields:

$$\frac{1}{2}\varepsilon_0\langle E^2\rangle = \frac{U_{m_ec^2}}{2V} \quad (4c)$$

Substituting $U_{m_ec^2} = hc/\lambda_C$ and $V = \pi\lambda^3/6$, we get:

$$\langle E^2 \rangle = \frac{U_{m_e c^2}}{\varepsilon_0 V} = \frac{hc}{\varepsilon_0 \lambda_C} \cdot \frac{6}{\pi \lambda_C^3} = \frac{6hc}{\pi \varepsilon_0 \lambda_C^4} \tag{5}$$

Taking the square root gives the average electric field magnitude inside the confined-photon electron model:

$$\langle E \rangle = \sqrt{\langle E^2 \rangle} = \sqrt{\frac{6hc}{\pi \varepsilon_0 \lambda_C^4}}. \tag{5a}$$

This corresponds to Equation (3) in WvdM [1]. (Plugging in constants yields an average electric field strength of $\langle E \rangle \approx 3.516 \times 10^{16}\ V/m$.)

2.4. Effective Radial Field & Gauss's Law

Given the double rotation of the electron structure, the (positive polarity of the) photon's electric field is directed radially inward (for the electron) everywhere on the toroidal surface — or outward (for the positron) — resembling a Coulomb field beyond the particle. To calculate the resulting charge, we evaluate the field at the characteristic radius of energy transport, defined previously as $R = \lambda_C/4\pi$. This radius marks the average path of the photon around its internal, toroidal momentum topology. It therefore represents the average boundary between the photon topology's inwardly directed and outwardly directed electric field polarities (following Gauss's law for non-spherical distributions [1,3]).

If we then imagine a point charge, q, at the very center of the volume — at the geometric origin — we can calculate the Coulomb field between this point charge and the electric field at the radius of transport, R.

2.5. Matching to Coulomb's Law

According to Coulomb's law, a theoretical point charge, q, at the center would produce an electric field of magnitude:

$$E_{\text{Coulomb}}(r) = \frac{q}{4\pi \varepsilon_0 r^2}. \tag{6}$$

This is also Equation (4) in WvdM [1]. In this model, the actual field at radius, r, (the mean radius of energy transport, which is our major toroidal radius, R) is roughly the average field $\langle E \rangle$, directed radially. Setting the model's field equal to the Coulomb field, we solve for the effective charge at radius, R, as

$$q \approx 4\pi\varepsilon_0 R^2 \langle E \rangle. \tag{6a}$$

(The geometry and energy localization justify treating $\langle E \rangle$ as representative of the field on the "mean transport" sphere [1], i.e., it is an effective coarse-grained matching rather than a pointwise identity.) Substituting the expressions for R and $\langle E \rangle$ (Equation (2) and (5a) above) yields:

$$q' = 4\pi\varepsilon_0 (\frac{\lambda_C}{4\pi})^2 \cdot \sqrt{\frac{6hc}{\pi\varepsilon_0 \lambda_C^4}} = \frac{\varepsilon_0 \lambda_C^2}{4\pi} \cdot \sqrt{\frac{6hc}{\pi\varepsilon_0 \lambda_C^4}}, \tag{6b}$$

which simplifies (note the change from h to $\hbar$, since $h = 2\pi\hbar$) to:

$$q' = \sqrt{\frac{3\varepsilon_0 hc}{8\pi^3}} = \frac{1}{2\pi}\sqrt{3\varepsilon_0 \hbar c} \tag{7}$$

where ε_0 is the vacuum permittivity, $\hbar$ is the reduced Planck's constant, and c is the speed of light in vacuum. **This is the Williamson-van der Mark expression for the electron's charge**.

Note carefully that, since the wavelengths cancel, the charge of the electron is *topological*, and therefore *independent of its wavelength, and thus, of its energy*.

If the values for the constants [4] are plugged in, the Williamson-van der Mark equation computes an 'idealized' elementary charge of:

$$q' = \frac{1}{2\pi}\sqrt{3\varepsilon_0 \hbar c} \approx 1.4584903365 \times 10^{-19} C \tag{7a}$$

While this result, $q' \approx 0.91e$, has the right order of magnitude, it contains an error of about 9 % from the observed value of the elementary charge, e.

According to Williamson, communicated in conversation with this author, this result is based upon the case where the path lengths of both photon loops are roughly equal to $\lambda_C/2$. However, as the two path lengths vary — which would also change the radius of primary energy transport — the calculated charge will increase, even up to an order of magnitude larger in the extremal case. (This is alluded to in the 1997 paper [1].)

3. REFINING THE MODEL

At the Quantum Bicycle Society — so named after the electron's double-loop, bi-cycle rotation — we have continued the work of our colleagues and mentors, John Williamson and Martin van der Mark. We will soon be releasing a series of papers that extend their brilliant

model by treating the electron geometry in a non-idealized manner. The results have been encouraging, and we look forward to releasing them. (Watch this space.)

These results include:

- Refining the WvdM charge equation to reflect an exact, quantized momentum geometry.
- Identifying the exact harmonic eigenmode that corresponds to the electron charge to within 1ppm.
- Identifying a closed second-order form of the electron's anomalous magnetic moment, agreeing to within 0.003% with the observed value.
- Identifying a fields-only mechanism for photon-to-electron particle formation and self-confinement.

REFERENCES

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